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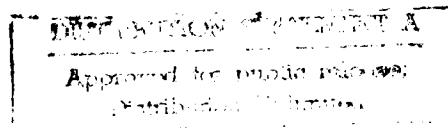
# Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program

Faydor L. Litvin and Jiao Zhang

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# Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program

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## Abstract

The contents of this report covers: (i) development of optimal geometry for crowned spur gears; (ii) methods for their generation; and (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned spur gears.

The developed method for synthesis is used for the determination of the optimal geometry for crowned pinion surface and is directed to reduce the sensitivity of the gears to misalignment, localize the bearing contact, and guarantee the favorable shape and low level of the transmission errors.

A new method for the generation of the crowned pinion surface has been proposed. This method is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. The crowned pinion surface can be also generated by a generating plane whose motion is provided by an automatic grinding machine controlled by a computer.

The TCA program simulates the meshing and bearing contact of the misaligned gears. The transmission errors are also determined.

## Summary

Spur gears are widely used to transmit mechanical power between parallel axes of drive shafts. Namely, such gears are used in planetary trains of helicopters. It is well known that spur involute gears are very sensitive to gear misalignments. Misalignment will cause: the shift of the bearing contact toward the edge of the gear tooth surfaces and transmission errors that increase gear noise. The previous methods for crowning of pinion tooth surface have been directed at providing of a favorable bearing contact. The transmission errors of the misaligned gears have been ignored. The new approach developed by the authors of this report is directed at the synthesis of spur involute gears with a crowned pinion surface that provides: (i) a parabolic type of low transmission errors for the misaligned gears and (ii) a localized bearing contact. New methods for the generation of crowned pinion surface proposed by the authors can be readily applied by the gear industry.

### List of Symbols

$a^*$	half the length of major axis of contact ellipse
$a_1$	constant
$a_i$	auxiliary vector in coordinate system i
$b$	dedendum of cutting tool
$d_1$	constant
$b^*$	half the length of minor axis of contact ellipse
$[D_{ij}]$	3x3 auxiliary matrix
$\zeta$	Operating center distance
$c$	operating center distance vector
$C^0 = \frac{N_1}{2P} + \frac{N_2}{2P}$	nominal center distance
$\tilde{C}^0$	nominal center distance vector
$u$	height of generating cone
$e_I^{(i)}, e_{II}^{(i)}$	unit vectors along principal direction of surface $\Sigma_i$ if principal directions of two surface coincide, i is neglected
$e_1$	constant
$f_i(x_1, x_2 \dots)$	function expression with respect to variable $x_1, x_2 \dots$
$F_1(x_1, x_2 \dots)$	function expression with respect to variable $x_1, x_2$
$f_i = \kappa_I^{(i)} - \kappa_{II}^{(i)}$	auxiliary function
$g(\phi_1)$	auxiliary function of $\phi_1$
$i_t, j_t, k_t$	unit direction vectors of coordinate system t
$\kappa_i$	auxiliary function
$L_{ij}$	line of tangency of surfaces $\Sigma_i$ and $\Sigma_j$
$[L_{ij}]$	projection transformation matrix (from $S_j$ to $S_i$ )

$m_1$	auxiliary function
$m_{1P} = \frac{d\phi_P}{ds}$	kinematic ratio
$m_{1P}' = \frac{dm_{1P}}{ds}$	derivative of kinematic ratio of gear and rack cutter
$m_{21} = \frac{d\phi_2}{d\phi_1}$	kinematic ratio of gears
$m_{21}' = \frac{dm_{21}}{d\phi_1}$	derivative of kinematic ratio of gears
$M_i$	point i
$[M_{ij}]$	coordinate transformation matrix (from $S_j$ to $S_i$ )
$\hat{n}_r$	velocity of end point of unit normal corresponding a point moving over a surface
$\hat{n}_i^{(j)}$	unit normal vector of surface $\Sigma_i$ in coordinate system $S_j$ [sometimes i or (j) is omitted if it is unnecessary to specify coordinate system]
$N_1$	number of pinion teeth
$N_2$	number of gear teeth
$O_i$	origin of coordinate system i
$P$	auxiliary function
$P$	diametral pitch
$Q$	auxiliary function
$r_1$	radius of pitch circle for pinion
$r_2$	radius of pitch circle for gear
$R_1$	radius of rotation to generate pinion tooth surface from a planar shape
$r_i^{(j)} (u, \theta)$	position vector describing surface $\Sigma$ with surface coordinate $(u, \theta)$ in coordinate system $S_i$ [sometimes (j) is omitted]
$s$	parameter of cutting motion
$S_i(x_i, y_i, z_i)$	coordinate system i

$u_i$	generating surface
$u_p^*$	auxiliary variable
$\tilde{v}_r$	velocity of a point moving over a surface
$\tilde{v}_j^{(i)}$	velocity of surface $\Sigma_i$ in coordinate system j (j is omitted sometimes)
$v_f^{(ij)} = \tilde{v}_f^{(i)} - v_f^{(j)}$	relative velocity between surface $\Sigma_i$ and $\Sigma_j$ at contact point in coordinate system f (f is omitted sometimes).
$v_I^{(ij)}, v_{II}^{(ij)}$	projection of $v^{(ij)}$ in the principal direction of surface
$x_i^{(j)}, y_i^{(j)}, z_i^{(j)}$	coordinates of vector in $S_i$ [sometimes (j) is omitted]
$\alpha$	half of cone angle
$\beta$	generating surface coordinate
$\Delta\gamma$	crossing angle between axes of pinion and gear
$\Delta\delta$	intersecting angle between axes of pinion and gear
$\Delta\phi_2$	kinematic error
$\Delta\phi_2'$	derivative of kinematic error with respect to $\phi_1$
$\varepsilon$	elastic deformation of the contacting point
$\vartheta_i$	generating surface coordinate
$\kappa_I^{(i)}, \kappa_{II}^{(i)}$	principal curvatures of surface $\Sigma_i$
$\kappa_\varepsilon^{(i)} = \kappa_I^{(i)} + \kappa_{II}^{(i)}$	auxiliary function
$\lambda$	generating surface coordinate
$\rho$	radius of genetrix arc for revolute surface
$\sigma^{(ij)}$	angle between $e_I^{(i)}$ and $e_I^{(j)}$
$\Sigma_1$	pinion tooth surface
$\Sigma_2$	gear tooth surface (or shape)
$\Sigma_g$	gear generating surface

$\Sigma_p$	pinion generating surface
$\phi_p$	angle of rotation for pinion being generated
$\phi_G$	angle of rotation for gear being generated
$\phi_1$	angle of pinion rotation in meshing with gear
$\phi_2$	angle of gear rotation in meshing with pinion
$\phi_{20}$	theoretical angle of gear rotation in meshing with pinion
$\psi_c$	pressure angle
$\omega^{(1)}$	magnitude of $\omega^{(1)}$
$\omega^{(2)}$	magnitude of $\omega^{(2)}$
$\tilde{\omega}$	angular velocity of pinion being generated
$\tilde{\omega}^{(2)}$	angular velocity of gear in meshing with
$\tilde{\omega}^{(12)} = \omega^{(1)} - \omega^{(2)}$	$\tilde{\omega}^{(12)} = \omega^{(1)} - \omega^{(2)}$
	relative angular velocity

## 1. Introduction

Spur involute gears are very sensitive to gear misalignments that cause: (i) the shift of the bearing contact to the edge of the gear tooth surfaces and (ii) transmission errors that increase gear noise. Many efforts have been done to improve the bearing contact of misaligned spur gears by crowning of pinion tooth surface. Wildhaber (1992) has proposed various methods of crowning that can be achieved in the process of gear generation. Maag engineers (see References) have used crowning as (i) longitudinal corrections (Fig. 1.1a), (ii) modified involute tooth profile uniform across facewidth (Fig. 1.1b), (iii) combination of longitudinal correction and uniform modified profile (Fig. 1.1c) and (iv) topological modification (Fig. 1.1d) that can provide any deviation of the crowned tooth surface from a regular involute surface.

The main purpose of the previously proposed methods for crowning was to improve the bearing contact of the misaligned gears. This resulted in only part of the solution. The transmission errors of the misaligned spur gears (a main source of the noise), were ignored. The influence of gear misalignment on the transmission errors has not been investigated. Maag's method of topological modification does not quantify the relationship between the surface deviation and the transmission error. Also the optimal geometry for the crowned pinion surface has not been proposed.

The contents of this report cover the solutions to the following problems: (i) optimal geometry of a crowned pinion tooth surface; (ii) new method of crowning, based on application of a tool with a surface of revolution (the tool surface is slightly deviated from a

cone surface) (iii) the development of Tooth Contact Analysis (TCA) programs for the determination of transmission errors for misaligned gears and their bearing contact.

The development of the optimal geometry of a crowned pinion gear tooth surface is based on the following considerations:

(i) Misaligned spur gears with a crowned pinion tooth surface can provide transmission errors  $\Delta\phi_2(\phi_1)$  of two types that are shown in Fig. 1.2a and 1.2b, respectively. The transmission errors are determined with the equation as

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2} \phi_1 \quad (1.1)$$

where:  $N_1$  and  $N_2$  are the numbers of gear teeth;  $\phi_1$  and  $\phi_2$  are the angles of gear rotation;  $\phi_2(\phi_1)$  is the function that relates the angles of rotation of gear and pinion if the pinion is crowned and the gears are misaligned;  $\phi_{20} = \frac{N_1}{N_2} \phi_1$  is the theoretical relation between the angles of rotation of the gears in the ideal case where the gears are not crowned, not misaligned and the transmission errors do not exist. Type 1 of transmission errors are not acceptable because the change of tooth meshing is accompanied with an interruption or interference of tooth surfaces. Type 2 of transmission errors is more preferable if the level of the transmission errors does not exceed a prescribed limit.

(ii) On the first glance crowning should be directed at providing an exact involute shape in the middle cross-section (Fig. 1.3). In reality this type of crowning is not acceptable because the misaligned gears will transform rotation with transmission

errors of type 1 (Fig. 1.2a). This is the reason why the authors have decided to synthesize a specific crowned pinion tooth surface. Such a pinion can provide transformation of rotation with a parabolic type of transmission error function when it is in meshing with a gear with a regular involute tooth surface. This type of function of transmission errors is synthesized for ideal gears that do not have any misalignment. Then, the tendency to provide parabolic transmission errors can be extended for the misaligned gears and the discontinuance of meshing can be avoided. It is necessary to emphasize that the proposed method of synthesis provides a shape in the middle cross-section of the pinion tooth that deviates in a certain way from the involute curve that is shown in Fig. 1.3. The longitudinal deviation from a straight line is not related with the transmission errors but with the desired dimensions of the instantaneous contact ellipse for the gear tooth surfaces. The proposed pinion tooth surface can be generated by a plane as the chosen generating surface. The motions of the plane with the respect to pinion must be controlled by a computer.

(iii) Another method of pinion crowning is based on application of a surface of revolution that slightly deviates from a regular tool conical surface (Fig. 1.4). Such a tool can be used as a grinding wheel or as a shaver. The motions of the tool and the gear being generated are related similarly to the motions of a rack-cutter and the gear (see Chapter 4). A tool with a regular conical surface can generate a crowned pinion tooth surface whose middle cross-section represents an involute curve (Fig. 1.3). However, this type of a crowned pinion tooth surface is not desirable because the misaligned gears provide transmission errors of type 1 (Fig. 1.2a). This is the

reason why a tool with the surface of revolution is to be used instead of a conical surface.

(iv) The evaluation of the bearing contact and transmission errors for the misaligned gears as well as the investigation of the influence of errors of gear assembly requires application of TCA programs. Such programs have been developed by the authors. The programs are based on the following algorithms:

(a) The contacting gear tooth surfaces are represented in a fixed coordinate system,  $S_f$ , that is rigidly connected to the gear housing (fig. 1.5).

(b) The continuous tangency of gear tooth surfaces is provided if the position-vectors and surface unit normals for the contacting surfaces coincide at the contact point at any instant. Then, we are able to determine the path of contact on the gear tooth surfaces and the relations between the angles of rotation of the output and input gears. Knowing function  $\phi_2(\phi_1)$ , we can determine the deviations of this function from the prescribed linear function i.e. the transmission errors.

(c) Due to the elasticity of the gear tooth surfaces the surface contact is spread over an elliptical area. The dimensions and orientation of the instantaneous contact ellipse depend on the principal curvatures and principal directions of the contacting tooth surfaces. The bearing contact is determined by the developed TCA program as the set of the contact ellipses that move over the contacting surfaces in the process of motion.

## 2. Optimal Pinion Geometry

### 2.1 Basic Consideration

It is assumed that only the pinion will be provided with a crowned tooth surface while the gear is provided with a regular involute surface. The crowned pinion tooth surface deviates from a regular involute surface. The topology of the crowned pinion tooth surface must satisfy the following requirements: (i) the pinion tooth surface is a continuous one; (ii) the pinion and gear tooth surfaces are in contact at a point at any instant; (iii) the principal directions and curvatures of the contacting surfaces must be related in such a way that the instantaneous contact ellipse will be of appropriate dimensions; (iv) the function of transmission errors of the misaligned gears should be of a parabolic type and its level must not extend the prescribed limits. The satisfaction to the described conditions is considered in the following sections of this part.

### 2.2 Parabolic Type of Function of Kinematic Errors

Initially we will consider that the gears are not misaligned, the pinion tooth surface is crowned and the gear tooth surface is a regular involute surface. The tooth surfaces are in point contact at every instant and the points of contact lie in the middle cross-section. Thus, initially we consider the meshing of gear tooth surfaces as a planar gearing. However, when simulating the meshing of gears by FCA program, we will consider the mesh of them as spatial gears.

Figure 2.2.1 shows the shapes of the gears which are in contact at a point that lies on the center distance. The shape of the gear

tooth profile is a regular involute curve, and the shape of the pinion tooth profile is to be determined. It is given that the shapes must provide transformation of rotation represented by the function (Fig. 2.2.2a)

$$\phi_2(\phi_1) = \frac{N_1}{N_2} \phi_1 + \Delta\phi_2(\phi_1) \quad (2.2.1)$$

where  $\Delta\phi_2(\phi_1)$  is a function of a parabolic type that represents the function of transmission errors;  $N_1$  and  $N_2$  are the numbers of gear teeth. We have to emphasize that the proposed method for synthesis provides low transmission errors for aligned gears and the function of transmission errors is a parabolic one. Then, the misaligned gears will keep the tendency to transform rotation with the same type of transmission errors and the discontinuance of contact by meshing will be avoided. This means that we have to avoid the appearance of transmission errors that are shown in Fig. 1.2a. Function  $\phi_2(\phi_1)$  contains a linear function represented by  $\frac{N_1}{N_2} \phi_1$  that relates the angles of rotation of the "ideal" input and output gears. Figure 2.2.2a shows the function  $\phi_2(\phi_1)$ . The function of transmission errors is represented to a larger scale in Fig. 2.2.2b as a periodic function of a parabolic type. The derivative

$$\Delta\phi'_2(\phi_1) = \frac{d}{d\phi_1} (\Delta\phi_2(\phi_1)) \quad (2.2.2)$$

is shown in Fig. 2.2.2c. For a "pure" parabolic function this derivative is a linear function. It is evident that the second derivative represented by

$$\frac{d^2}{d\phi_1^2} (\Delta\phi_2(\phi_1)) = \frac{d^2}{d\phi_1^2} (\phi_2(\phi_1))$$

is negative. The transformation of rotation by the gears will be performed with a non-constant gear ratio

$$m_{21} = \frac{\omega_2}{\omega_1} = \frac{N_1}{N_2} + \Delta\phi_2'(\phi_1) \quad (2.2.3)$$

The instantaneous center of rotation I is moved along the gear center distance in the process of gear rotation. Figure 2.2.1 shows the instantaneous location of I.

More details about the geometry of the pinion that corresponds to the discussed function  $\phi_2(\phi_1)$  will be given in Chapter 3.

### 2.3 Relation Between the Principal Curvatures and Directions

The relation between the principal curvatures and directions of the contacting surfaces and the derivative of the gear ratio  $m_{21}'$  is represented as follows (Litvin, 1987)

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \quad (2.3.1)$$

For the case of spur gears with the crowned pinion tooth surface the elements of the determinant of Eq. (2.3.1) are represented by the following equations

$$v_{21} = v_{12} \quad b_{31} = b_{13} \quad b_{32} = b_{23} \quad (2.3.2)$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa_I^{(2)} & 0 \\ 0 & -\kappa_{II}^{(1)} + \kappa_{II}^{(2)} \end{bmatrix} \quad (2.3.3)$$

$$\begin{bmatrix} v_{13} \\ v_{23} \end{bmatrix} = \begin{bmatrix} -e_I \cdot \omega^{(12)} \\ e_{II} \cdot \omega^{(12)} \end{bmatrix} \quad (2.3.4)$$

$$v_{33} = -n \cdot i(\omega^{(1)} \times v_{\text{tr}}^{(2)}) - (\omega^{(2)} \times v_{\text{tr}}^{(1)}) \\ + (\omega^{(1)})^2 m_{21}^2 (n \times \kappa_2) \cdot (r_f^{(1)} - c) \quad (2.3.5)$$

Figure 2.3.1 shows the tangent plane to the gear tooth surfaces at the point of contact I (see Fig. 2.2.1). The principal directions are represented by the unit vectors  $\tilde{e}_I$  and  $\tilde{e}_{II}$  that are the same for the both contacting surfaces. Vector  $\tilde{e}_I$  is shown in Fig. 2.3.1 and is represented in coordinate system  $S_f$  as follows

$$\tilde{e}_I = \begin{bmatrix} \sin \psi_C \\ -\cos \psi_C \\ 0 \end{bmatrix} \quad (2.3.6)$$

where  $\psi_C$  is the pressure angle at point I. Vector  $\tilde{e}_{II}$  is directed parallel to the negative  $z_f$ -axis and is represented by

$$\tilde{e}_{II} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad (2.3.7)$$

The surface unit normal vector is represented by (Fig. 2.2.1)

$$\tilde{n}_f = \tilde{e}_I \times \tilde{e}_{II} = \begin{bmatrix} \cos \psi_c \\ \sin \psi_c \\ 0 \end{bmatrix} \quad (2.3.8)$$

The angular velocities of rotation are represented by (Fig. 2.2.1)

$$\tilde{\omega}^{(1)} = \omega^{(1)} \begin{bmatrix} u \\ 0 \\ -1 \end{bmatrix} \quad \tilde{\omega}^{(2)} = \omega^{(2)} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (2.3.9)$$

Vectors  $\tilde{v}_{tr}^{(1)}$  and  $\tilde{v}_{tr}^{(2)}$  represent the velocities of the contact point in transfer motion, while gears rotate. Thus (Fig. 2.2.1)

$$\tilde{v}_{tr}^{(1)} = \omega^{(1)} O_1 I \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \tilde{v}_{tr}^{(2)} = \omega^{(2)} O_2 I \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2.3.10)$$

Vector  $\tilde{c}$  and  $\tilde{r}_f^{(1)}$  are

$$\tilde{c} = v_1 u_2 \begin{bmatrix} u \\ 1 \\ v \end{bmatrix} \quad \tilde{r}_f^{(1)} = O_1 I \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (2.3.11)$$

where  $\psi_1 \psi_2 = C^0 = \frac{N_1}{2p} + \frac{N_2}{2p}$ ,  $\psi_1 I = \frac{N_1}{2p}$ ,  $\psi_2 I = \frac{N_2}{2p}$ ;  $p$  is diametral pitch and  $C^0$  is ideal or nominal central distance.

$\kappa_I^{(2)}$  and  $\kappa_{II}^{(2)}$  are the first and the second principal curvatures of the gear tooth surface represented by

$$\kappa_I^{(2)} = \frac{1}{\psi_2 I \sin \psi_C} \quad \kappa_{II}^{(2)} = 0 \quad (2.3.12)$$

where  $\psi_2 I = \frac{N_2}{2p}$  is the radius of the gear pitch circle. The positive sign of the curvature indicates that the curvature center lies on the positive direction of the surface unit normal. The unit vector  $\hat{k}_2$  in Eq. (2.3.5) coincides with the unit vector of  $\omega^{(2)}$ . Thus  $\hat{k}_2 = \hat{k}_f$  where  $\hat{k}_f$  is the unit vector of the  $z_f$ -axis (Fig. 2.2.1).

Equations (2.3.1) - (2.3.12) yield the following relation between the derivative  $m'_{21} = \frac{d}{d\phi_1} (m_{21}(\phi_1))$  and the principal curvatures of the crowned pinion tooth surface

$$m'_{21} = - \frac{A}{B}$$

where

$$A = |\kappa_I^{(1)}| - \frac{2p}{N_1 \sin \psi_C}$$

$$B = \frac{N_2}{N_1 \tan \psi_C} \left[ \frac{2p}{N_1 \sin \psi_C} + \frac{N_2 (|\kappa_I^{(1)}| - \frac{2p}{N_1 \sin \psi_C})}{N_1 + N_2} \right] \quad (2.3.13)$$

The goal is to provide a parabolic type of the transmission error function at least in the neighborhood of the main contact point I.

This requirement can be satisfied with the negative value of  $m'_{21}$ . Using Eq. (2.3.13), we can control the value of  $m'_{21}$  (it is negative) by varying the values of  $\kappa_I^{(1)}$ . It is evident that  $m'_{21}$  is negative if  $|\kappa_I^{(1)}| > \frac{2p}{N_1 \sin \psi_C}$ . Here:  $\frac{2p}{N_1 \sin \psi_C}$  is the curvature at point I if the pinion would be provided with a regular involute curve;  $\kappa_I^{(1)}$  is the curvature of the modified shape of the pinion tooth in its middle cross-section. We have emphasized that the pinion tooth surface must be deviated in the longitudinal direction (Fig. 1.3) to provide the point contact of the mating tooth surfaces and localize the bearing contact.

### 3. Design of the Crowned Pinion Tooth Surface for the Prescribed Transmission Error Function

#### 3.1 Basic Concept

Consider that the function  $\phi_2(\phi_1)$  that relates the angles of rotation of the gears is represented by (Fig. 3.3.1).

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + b_1 - a_1 \phi_1^2 \quad (3.1.1)$$

Here:  $\phi_1 \frac{N_1}{N_2}$  is the linear part of function  
 $\phi_2(\phi_1)$  and  $\Delta\phi_2(\phi_1) = b_1 - a_1 \phi_1^2$  is prescribed parabolic function of transmission errors.

The gear is provided with a regular involute surface. If the gears are aligned, the crowned pinion tooth surface and the gear tooth surface are in mesh in the middle cross-section. The contact of the gear tooth surface can be considered as the contact of two planar shapes. Our goal is to determine the shape of the pinion tooth surface in the middle cross-section.

We consider that the transmission errors in the region of

$-\frac{\pi}{N_1} < \phi_1 < \frac{\pi}{N_1}$  are on average equal to zero. Thus

$$\int_{-\frac{\pi}{N_1}}^{\frac{\pi}{N_1}} (b_1 - a_1 \phi_1^2) d\phi_1 = 0 \quad (3.1.2)$$

Equation (3.1.2) yields that

$$a_1 = \frac{c_1}{(\frac{\pi}{N_1})^2} \quad b_1 = \frac{c_1}{3} \quad (3.1.3)$$

Henceforth we will use the following coordinate systems (Fig. 3.1.2):  $S_1(x_1, y_1)$  and  $S_2(x_2, y_2)$  that are rigidly connected to the pinion and gear respectively; and  $S_f(x_f, y_f)$  that is the fixed coordinate system employed in the housing.

The determination of the pinion tooth shape is based on the following considerations:

(i) The gear tooth shape  $\Sigma_2$  is a regular involute curve represented in coordinate system  $S_2$  by the vector-function  $r_2(\phi_G)$ ; the shape unit normal is represented by  $n_2(\phi_G)$ .

(ii) using the coordinate transformation in transition from  $S_2$  to  $S_f$ , we can represent the family of curves  $\Sigma_G$  in  $S_f$  as follows

$$[r_f^{(2)}] = [M_{f2}] [r_2] \quad [n_f^{(2)}] = [L_{f2}] [n_2] \quad (3.1.4)$$

(iii) The equation of meshing is represented by

$$n_f^{(2)} \cdot v_f^{(21)} = 0 \quad (3.1.5)$$

where  $v_f^{(21)}$  is the sliding velocity at the contact point.

Equations (3.1.4) and (3.1.5) yield the following relation

$$f(\phi_G, \phi_2) = 0 \quad (3.1.6)$$

that is called the equation of meshing.

(iv) The following equation

$$[r_1] = [M_{1f}] [r_f^{(2)}] = [M_{1f}] [M_{f2}] [r_2] \quad (3.1.7)$$

and the equation of meshing (3.1.6) determine the shape of the pinion tooth in the middle cross-section. The surface of the pinion can be determined based on the shape in its middle cross-section.

### 3.2 Derivation of Pinion Tooth Surface

#### Gear Tooth Involute Shape Equations

The gear involute shape and its unit normal are represented in  $S_2$  as follows:

$$x_2 = r_2 [\sin \phi_G - \phi_G \cos \psi_C \cos(\phi_G - \psi_C)]$$

$$y_2 = r_2 [\cos \phi_G + \phi_G \cos \psi_C \sin(\phi_G - \psi_C)] \quad (3.2.1)$$

$$z_2 = 0$$

$$\mathbf{n}_2 = \begin{bmatrix} -\cos(\phi_G - \psi_C) \\ \sin(\phi_G - \psi_C) \\ 0 \end{bmatrix} \quad (3.2.2)$$

Here:  $r_2$  is the radius of the gear pitch circle,  $\psi_C$  is the pressure angle and  $\phi_G$  is the involute curve parameter;  $\mathbf{n}_2$  is the normal of tooth shape.

Matrix  $[M_{f2}]$  is represented by

$$[M_{f2}] = \begin{bmatrix} -\cos \phi_2 & \sin \phi_2 & 0 & 0 \\ -\sin \phi_2 & -\cos \phi_2 & 0 & C^0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2.3)$$

Using Eqs. (3.1.4), (3.2.1), (3.2.2) and (3.2.3), we represent the family of shapes  $\Sigma_2$  and the shape unit normal in  $S_f$  as follows

$$[r_f^{(2)}] = r_2 \begin{bmatrix} -\sin(\phi_G - \phi_2) + \phi_G \cos \psi_C \cos(\phi_G - \psi_C - \phi_2) \\ -\cos(\phi_G - \phi_2) - \phi_G \cos \psi_C \sin(\phi_G - \psi_C - \phi_2) \\ 0 \\ 1 \end{bmatrix} \quad (3.2.4)$$

$$[n_f^{(2)}] = \begin{bmatrix} \cos(\phi_G - \psi_C - \phi_2) \\ -\sin(\phi_G - \psi_C - \phi_2) \\ 0 \end{bmatrix} \quad (3.2.5)$$

### Equation of Meshing

The sliding velocity  $\tilde{v}^{(21)}$  is represented by the following equation

$$\tilde{v}^{(21)} = \tilde{\omega}^{(2)} \times (\overline{o_2 o}_1 + \tilde{r}_f^{(2)}) - \tilde{\omega}^{(1)} \times \tilde{r}_f^{(2)} \quad (3.2.6)$$

Here (Fig. 3.1.2)

$$\tilde{z}^{(1)} = \omega^{(1)} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad \overline{o_2 o}_1 = \begin{bmatrix} 0 \\ -c^o \\ 0 \end{bmatrix} \quad \tilde{\omega}^{(2)} = \omega^{(2)} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$c^o = r_2 \left( \frac{n_1 + n_2}{n_1} \right) \quad \omega_2 = \omega_1 \left( \frac{n_1}{n_2} \right) - 2a_1 \phi_1$$

The expression for  $\omega_2$  is obtained by differentiating Eq. (3.1.1). Equation (3.2.6) yields

$$\dot{z}^{(2)} = \begin{bmatrix} -y_f^{(2)} \\ x_f^{(2)} \\ 0 \end{bmatrix} (\omega^{(2)} - \omega^{(1)}) + \begin{bmatrix} c^o \\ 0 \\ 0 \end{bmatrix} \omega^{(2)} \quad (3.2.7)$$

Equations (3.1.5), (3.2.7), (3.2.4) and (3.2.5) yield the following equation of meshing

$$f(\phi_G, \phi_1) = \cos \psi_C \left( \frac{n_1}{n_2} + 1 - 2a_1 \phi_1 \right) - \left( \frac{n_1}{n_2} + 1 \right) \cos(\phi_G - \psi_C - \phi_2) = 0 \quad (3.2.8)$$

where  $\phi_2 = \frac{n_1}{n_2} \phi_1 + (\phi_1 - a_1 \phi_1^2)$ .

Fixing  $\phi_1$ , we can obtain parameter  $\phi_G$ . Equation (3.2.8) provides two solutions for  $\phi_G$ . Therefore it is necessary to choose the solution corresponding to the working part of shape  $\Sigma_2$ .

### Pinion Tooth Shape Equations

Matrix  $[M_{1f}]$  is represented by (Fig. 3.1.2)

$$[M_{1f}] = \begin{bmatrix} \cos \phi_1 & -\sin \phi_1 & 0 & 0 \\ \sin \phi_1 & \cos \phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2.9)$$

Equations (3.2.9), (3.1.7), (3.2.4) and (3.2.8) yield the following equations for the pinion shape

$$x_1 = -C^O \sin \phi_1 + r_2 \sin [\phi_1 - \psi_c + g(\phi_1)] \quad (3.2.10)$$

$$+ r_2 \left[ \frac{N_1}{N_2} \phi_1 + b_1 - a_1 \phi_1^2 + \psi_c - g(\phi_1) \right] \cos \psi_c \cos [\phi_1 + g(\phi_1)]$$

$$y_1 = C^O \cos \phi_1 - r_2 \cos [\phi_1 - \psi_c + g(\phi_1)]$$

$$+ r_2 \left[ \frac{N_1}{N_2} \phi_1 + b_1 - a_1 \phi_1^2 + \psi_c - g(\phi_1) \right] \cos \psi_c \sin [\phi_1 + g(\phi_1)]$$

$$z_1 = 0$$

Here:

$$g(\phi_1) = \arccos \left\{ \left[ 1 - \frac{2a_1 \phi_1}{(N_1/N_2) + 1} \right] \cos \psi_c \right\}$$

Equations (3.2.10) represent the pinion shape in parametric form, with the parameter  $\phi_1$ . For the case where the prescribed transmission errors are zero,  $a_1 = 0$ ,  $b_1 = 0$ ,  $g(\phi_1) = \psi_c$  and Eq. (3.2.10) represent the pinion tooth shape as a regular involute shape.

#### Pinion Tooth Surface Equations

The crowned pinion tooth surface must be deviated from a cylindrical surface in the longitudinal direction (Fig. 1.3). The

simplest way to obtain such deviation is based on representation of the pinion tooth surface as a surface of revolution. Such a surface can be generated by the rotation of the pinion tooth shape about a fixed axis. The determination of the pinion surface of revolution can be performed by using the following procedure:

(i) Assume that the pinion tooth shape has been transmitted from the coordinate system  $S_1$  to the auxiliary coordinate system  $S_a$  (Fig. 3.2.1a). Thus the tooth shape is now rigidly connected to coordinate system  $S_a$  and is represented in  $S_a$  by the following matrix equation:

$$[r_a] = [M_{a1}] [r_1(\phi_1)] \quad (3.2.11)$$

where

$$[M_{a1}] = \begin{bmatrix} 1 & 0 & 0 & R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2.12)$$

(ii) On the second stage we use a fixed coordinate  $S_b$  and rotate the coordinate system  $S_a$  with the tooth shape about the  $y_b$ -axis. The tooth shape will generate a surface of revolution represented in  $S_b$  as follows (Fig. 3.2.1b)

$$[r_b] = [M_{ba}] [r_a] \quad (3.2.13)$$

where

$$[M_{ba}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2.14)$$

(iii) Now, we can represent the generated pinion tooth surface in coordinate system  $S_1$  using the following matrix equation (Fig. 3.21c):

$$[r_1] = [M_{1b}] [r_b] \quad (3.2.15)$$

where

$$[M_{1b}] = \begin{bmatrix} 1 & 0 & 0 & -R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.2.16)$$

The final expression of the pinion tooth surface can be represented now as follows

$$[r_1(\theta, \phi_p)] = \begin{bmatrix} f_1(\theta, \phi_p) \\ f_2(\theta, \phi_p) \\ f_3(\theta, \phi_p) \\ 1 \end{bmatrix} = [M_{1b}] [M_{ba}(\theta)] [M_{a1}] [r_1(\phi_p)] \quad (3.2.17)$$

Equations (3.2.11) – (3.2.17) yield

$$f_1(\theta, \phi_p) = x_1(\phi_p)\cos\theta - R(1-\cos\theta)$$

$$f_2(\theta, \phi_p) = y_1(\phi_p)$$

$$f_3(\theta, \phi_p) = x_1(\phi_p)\sin\theta + R\sin\theta \quad (3.2.18)$$

Here  $x_1(\phi_p)$ ,  $y_1(\phi_p)$  are functions (Eq. 3.2.10) that represent the pinion shape in the middle cross-section. Also for convenience, we use  $\phi_p$  as curve parameter instead of  $\phi_1$ .

### 3.3 Simulation of Meshing and Bearing Contact

The authors have developed the TCA (Tooth Contact Analysis) program to simulate the meshing and bearing contact of misaligned gears (see the Appendix). The results of the TCA program confirm that the proposed method of crowning reduces the sensitivity of gears to the misalignment (as shown in numerical example).

#### Example

Given: numbers of teeth:  $N_1 = 20$ ,  $N_2 = 40$ ; diametral pitch  $P = 1\frac{1}{2}$  in; pressure angle  $\psi_c = 20^\circ$ . The pinion tooth surface has been designed as a crowned surface and the function of transmission errors for the gears without misalignments has been represented as a parabolic function with the "Level of Kinematical Errors (LKE)" = 2 arc second. Radius of rotation  $R_1 = 350$  in (see Fig. 3.2.1). The developed TCA program has been applied for the evaluation of transmission errors for the following misalignments:

- (i) The change of the center distance is  $\frac{\Delta C}{C} = 1\%$ . The gear axes are not parallel but crossed and the twist angle is 5 arc minutes. The

function of kinematic errors caused by the misalignments mentioned above is of a parabolic type and the maximum value of transmission errors is 1.2 arc seconds.

(ii) The gear axes are not parallel but intersected and form the angle  $\alpha = 5$  arc minutes. The function of kinematic errors is of a parabolic type and the maximum value of transmission errors is 2.0 arc seconds.

#### **4. Deviation of Pinion Tooth Surface by Tool with Cone or Revolute Surface**

##### **4.1 Introduction**

A new method for generation of the pinion of spur gearing is developed. The involute spur gears with the crowned pinion tooth surface are less sensitive to the gear misalignment, the bearing contact has a favorable location and the kinematical errors are very small and with favorable shape. The crowning may be done by grinding or shaving using a tool with conic surface, a surface of revolution or a planar surface. The new method covers: (i) the theory of the proposed method of crowning, (ii) determination of the pinion surface generated by the tool, (iii) conditions of non-undercutting, (iv) principal curvatures and directions of the pinion tooth surface, (v) dimensions and orientation of the instantaneous contact ellipse, (vi) the bearing contact and (vii) the kinematical errors due to gear misalignment. Computer programs are developed in all stages including the simulation of meshing and bearing contact TCA (Tooth Contact Analysis) program.

##### **4.2 Principle of Generation and Coordinate Systems**

Consider two rigidly connect generating surfaces  $\Sigma_G$  and  $\Sigma_p$ . The generating surface is a plane and generates the gear tooth surface  $\Sigma_2$  that is a regular involute surface. Surface  $\Sigma_p$  is a surface of revolution. Initially we consider that  $\Sigma_p$  is a cone surface and  $\Sigma_p$  and  $\Sigma_G$  contact each other along a straight line that is the generatrix of the cone.

Figure 4.2.0 shows the generating surfaces  $\Sigma_G$  and  $\Sigma_p$ . Figure 1.4

snows a sketch of the tool that could be used for pinion generation. Figure 4.2.1 illustrates the process of generation. While the rigidly connected generating surfaces perform a translational motion, the pinion and gear rotate about their axes  $O_1$  and  $O_2$  respectively. The parameter of motion of the cutters,  $s$ , and the angles of rotation of the pinion and the gear,  $\phi_p$  and  $\phi_G$ , are related as follows

$$s = r_1 \phi_p = r_2 \phi_G \quad (4.2.1)$$

where  $r_1$  and  $r_2$  are the gear centrododes radii; the cutter centrodode is the straight line that is tangent to the gear centrododes. Point I is the instantaneous center of rotation. Coordinate systems  $S_1$  and  $S_2$  are rigidly connected to the pinion and the gear respectively. The rigidly connected tool surfaces (see Fig. 4.2.0) are represented in coordinate system  $S_C$ ;  $S_f$  is the fixed coordinate system.

The generating surface  $\Sigma_G$  (a plane) is covered with the set of contact lines  $L_{G2}$  -- the instantaneous lines of tangency of surfaces  $\Sigma_G$  and  $\Sigma_2$  (Fig. 4.2.2a). The location of these lines depends on the value of parameter  $\phi_G$ . Line  $L_{GP}$  is the line of tangency of generating surfaces  $\Sigma_G$  and  $\Sigma_P$ . Points  $M_1, M_2, \dots, M_n$  represent the set of points of intersection of  $L_{GP}$  and  $L_{G2}$ . The coordinate transformation from  $S_C$  to  $S_2$  is represented by the equation

$$[r_2] = [n_{2C}] [r_C^{(G)}] \quad (4.2.2)$$

$$n^{(G)} \cdot v^{(G2)} = f_2(u_G, \alpha_G, \phi_G) = u \quad (4.2.3)$$

Here:  $\tilde{r}_c^{(G)}(u_G, \theta_G)$  is the vector function that represents plane  $\Sigma_G$ ;  $[M_{2C}]$  is the matrix that describes the coordinate transformation from  $S_C$  to  $S_2$ . Equation (4.2.3) is called the equation of meshing of the gear cutter and gear 2 [Litvin, 1987].  $\tilde{n}^{(G)}$  is the unit normal of generating surface and  $\tilde{v}^{(C2)} = \tilde{v}^{(C)} - \tilde{v}^{(2)}$  is the relative velocity for the process of generation. Contact point  $M_i$  of surfaces  $\Sigma_1$  and  $\Sigma_2$  may be represented in coordinate system  $S_C$  as a function of parameter  $\phi_G$  as follows

$$\begin{aligned}\tilde{r}_c^{(G)} &= \tilde{r}_c^{(G)}(u_G, \theta_G) & f_2(u_G, \theta_G, \phi_G) &= 0 \\ F_2(u_G, \theta_G) &= 0\end{aligned}\quad (4.2.4)$$

Equations

$$\tilde{r}_c^{(G)} = \tilde{r}_c^{(G)}(u_G, \theta_G), \quad F_2(u_G, \theta_G) = 0 \quad (4.2.5)$$

represent line  $L_{GP}$  in coordinate system  $S_C$ . Equations (4.2.4) represent the location of points  $M_1, M_2, \dots, M_n$  on surface  $\Sigma_G$  as a function of parameter  $\phi_G$ . Matrix Eq. (4.2.2) and Eq. (4.2.4) represent the set of contact points of gear tooth surfaces  $\Sigma_2$  and  $\Sigma_1$  in coordinate system  $S_2$ . These points are the centers of the instantaneous contact ellipses that form the bearing contact of gear 2 and pinion 1.

Using the matrix equation

$$[r_f] = [M_{2C}] [\tilde{r}_c^{(G)}] \quad (4.2.6)$$

and Eq. (4.2.4) we may represent in coordinate system  $S_f$  the line of action --- the set of contact points of  $\Sigma_1$  and  $\Sigma_2$  in the fixed coordinate system.

Similarly, considering the generating surface  $\Sigma_p$ , we may represent in coordinate system  $S_c$  the lines of contact,  $L_{p1}$ , of surfaces  $\Sigma_p$  and  $\Sigma_1$  as follows

$$\tilde{r}_c^{(p)} = \tilde{r}_c^{(p)}(u_p, \theta_p) \quad \tilde{n}^{(p)} \cdot \tilde{v}^{(p1)} = f_1(u_p, \theta_p, \phi_p) = 0 \quad (4.2.7)$$

here  $\tilde{n}^{(p)}$  is the unit normal of generating surface  $\Sigma_p$ ;  $\tilde{v}^{(p1)} = \tilde{v}^{(p)} - \tilde{v}^{(1)}$  is the relative velocity for the process of generation.

The pinion surface is represented by the equations

$$[r_1] = [M_{1c}] [r_c^{(p)}] = [M_{1f}] [M_{fc}] [r_c^{(p)}] \quad f_1(u_p, \theta_p, \phi_p) = 0 \quad (4.2.8)$$

#### Equations

$$\tilde{r}_c^{(p)} = \tilde{r}_c^{(p)}(u_p, \theta_p), \quad F(u_p, \theta_p) = 0 \quad (4.2.9)$$

represent, in terms of  $u_p$ , and  $\theta_p$ , the line of contact of surfaces  $\Sigma_p$  and  $\Sigma_G$ . in coordinate system  $S_c$ . Contact point  $M_i$  of surfaces  $\Sigma_1$  and  $\Sigma_2$  may be represented in coordinate system  $S_c$  as a function of parameter  $\phi_p$  as follows

$$\tilde{r}_c^{(p)} = \tilde{r}_c^{(p)}(u_p, \theta_p) \quad f_1(u_p, \theta_p, \phi_p) = 0 \quad (4.2.10)$$

$$F_1(u_p, \theta_p) = 0$$

using the matrix equation

$$[r_f] = [M_{fc}] [r_c^{(P)}] \quad (4.2.11)$$

and Eq. (4.2.10) we may represent the line of action of gears 1 and 2 as a function of  $\phi_p$  in coordinate system  $S_f$ . Equations (4.2.4) and (4.2.6), and (4.2.10) and (4.2.11), determine the same line of action but in different parameter,  $\phi_G$  or  $\phi_p$ , respectively.

#### 4.3 Tool Surface

The pinion generating surface is a cone and may be represented in an auxiliary coordinate system  $S_a$  as follows (Fig. 4.3.1)

$$\begin{aligned} x_a &= u_p \sin \alpha \cos \theta_p & y_a &= d - u_p \cos \alpha \\ z_a &= u_p \sin \alpha \sin \theta_p & 0 < u_p < \frac{d}{\cos \alpha} & 0 < \theta_p < 2\pi \end{aligned} \quad (4.3.1)$$

The surface normal is represented by

$$\hat{n}_a = \frac{\partial r_a}{\partial \theta_p} \times \frac{\partial r_a}{\partial u_p} = u_p \sin \alpha \begin{bmatrix} \cos \alpha \cos \theta_p \\ \sin \alpha \\ \cos \alpha \sin \theta_p \end{bmatrix} \quad (4.3.2)$$

The unit surface normal is (provided  $u_p \sin \alpha \neq 0$ )

$$[n_a] = \begin{bmatrix} \cos \alpha \cos \theta_p \\ \sin \alpha \\ \cos \alpha \sin \theta_p \end{bmatrix} \quad (4.3.3)$$

Figure 4.3.2 illustrates the installment of the tool cone in coordinate system  $S_C$ . Axes of coordinate system  $S_b$  are parallel to axes of system  $S_a$  and axes  $x_b, y_b$  lie in plane  $z_C = 0$ . The coordinate transformation from  $S_a$  to  $S_C$  is represented by the following matrix equation

$$[r_C^{(P)}] = [M_{cb}] [M_{ba}] [r_a] = [M_{ca}] [r_a] \quad (4.3.4)$$

here

$$[M_{cb}] = \begin{bmatrix} \cos(\alpha - \psi_C) & \sin(\alpha - \psi_C) & 0 & 0 \\ -\sin(\alpha - \psi_C) & \cos(\alpha - \psi_C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3.5)$$

$$[M_{ba}] = \begin{bmatrix} 1 & 0 & 0 & -u_P^* \sin \alpha \\ 0 & 1 & 0 & -(d - u_P^* \cos \alpha) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3.6)$$

Combining Eq. (4.3.5) and Eq. (4.3.6) results in:

$$[M_{ca}] = \begin{bmatrix} \cos(\alpha - \psi_C) & \sin(\alpha - \psi_C) & 0 & -u_P^* \sin \psi_C - d \sin(\alpha - \psi_C) \\ -\sin(\alpha - \psi_C) & \cos(\alpha - \psi_C) & 0 & u_P^* \cos \psi_C - d \cos(\alpha - \psi_C) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3.7)$$

$$\text{Here } u_p^* = |\overline{AO}_c| = \frac{d}{\cos \alpha} - \frac{b}{\cos \psi_c}$$

Using Eqs. (4.3.4), (4.3.7) and (4.3.1), we represent the generating surface  $\Sigma_p$  in coordinate system  $S_c$  by equations as follows

$$[r_c^{(p)}] = \begin{bmatrix} u_p [\sin \alpha \cos \theta_p \cos(\alpha - \psi_c) - \cos \alpha \sin(\alpha - \psi_c)] - u_p^* \sin \psi_c \\ -u_p [\sin \alpha \cos \theta_p \sin(\alpha - \psi_c) + \cos \alpha \cos(\alpha - \psi_c)] + u_p^* \cos \psi_c \\ u_p \sin \alpha \sin \theta_p \\ 1 \end{bmatrix} \quad (4.3.8)$$

It is evident that the point of the cone surface with the surface parameter  $u_p = u_p^*$  and  $\theta_p = 0$  coincides with the origin  $O_c$  of coordinate system  $S_c$ . The unit normal of cone surface is represented in coordinate system  $S_c$  by

$$[n_c^{(p)}] = [L_{ca}] [n_a] \quad (4.3.9)$$

here

$$[L_{ca}] = \begin{bmatrix} \cos(\alpha - \psi_c) & \sin(\alpha - \psi_c) & 0 \\ -\sin(\alpha - \psi_c) & \cos(\alpha - \psi_c) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.3.10)$$

Equations (4.3.9), (4.3.10) and (4.3.3) yield

$$[n_c^{(P)}] = \begin{vmatrix} \cos\alpha \cos\theta_P \cos(\alpha - \psi_C) + \sin\alpha \sin(\alpha - \psi_C) \\ -\cos\alpha \cos\theta_P \sin(\alpha - \psi_C) + \sin\alpha \cos(\alpha - \psi_C) \\ \cos\alpha \sin\theta_P \end{vmatrix} \quad (4.3.11)$$

#### 4.4 Pinion Tooth Surface

The equation of meshing of the generating surface  $\Sigma_P$  and the pinion tooth surface is represented by

$$\underline{n}^{(P)} \cdot \underline{v}^{(P1)} = \underline{n}^{(P)} \cdot (\underline{v}^{(P)} - \underline{v}^{(1)}) = 0 \quad (4.4.1)$$

We may also use the equation based on the condition that the contact normal must intersect the instantaneous axis of rotation I-I (Litvin, 1987). Thus we obtain (Fig. 4.2.1)

$$\frac{x_c - x_c^{(P)}}{n_{cx}^{(P)}} = \frac{y_c - y_c^{(P)}}{n_{cy}^{(P)}} = \frac{z_c - z_c^{(P)}}{n_{cz}^{(P)}} \quad (4.4.1a)$$

Here:  $(x_c, y_c, z_c)$  are the coordinates of a point that lies on axis I-I;  $x_c^{(P)}, y_c^{(P)}$  and  $z_c^{(P)}$  are the coordinates of the cone surface;  $n_{cx}$ ,  $n_{cy}$  and  $n_{cz}$  are the projections of the surface unit normal. Here (Fig. 4.2.1)

$$x_c = s = r_1 \phi_P \quad y_c = 0 \quad (4.4.2)$$

Equations (4.4.1a), (4.4.2), (4.3.8) and (4.3.11) yield

$$f_1(u_p, \theta_p, \phi_p) = r_1 \phi_p [-\cos \alpha \cos \theta_p \sin(\alpha - \psi_c) + \sin \alpha \cos(\alpha - \psi_c)]$$

$$-u_p \cos \theta_p + u_p^* (\cos \theta_p \cos^2 \alpha + \sin^2 \alpha) = 0 \quad (4.4.3)$$

here (Fig. 4.3.2)

$$u_p^* = AO_c = \frac{d}{\cos \alpha} - \frac{b}{\cos \psi_c} \quad (4.4.4)$$

The line of contact  $L_{PG}$  of generating surfaces  $\Sigma_p$  and  $\Sigma_G$  (Fig. 4.2.0) is generatrix of the cone surface determined with

$$\theta_p = 0 \quad (4.4.5)$$

Equations (4.3.8), (4.4.3) and (4.4.5) represent the location of the instantaneous contact point of surfaces  $\Sigma_1$  and  $\Sigma_2$  on surface  $\Sigma_p$  as follows

$$x_c^{(P)} = r_1 \phi_p \sin^2 \psi_c \quad y_c^{(P)} = -r_1 \phi_p \sin \psi_c \cos \psi_c \quad z_c^{(P)} = 0 \quad (4.4.6)$$

The coordinate transformation in transition from  $S_c$  to  $S_f$  is represented by the matrix equation

$$[r_f] = [M_{fc}] [r_c^{(P)}] \quad (4.4.7)$$

where (Fig. 4.2.1)

$$[M_{fc}] = \begin{bmatrix} 1 & 0 & 0 & -r_1 \phi_p \\ 0 & 1 & 0 & r_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.4.8)$$

Equations (4.4.6) to (4.4.8) yield

$$x_f = r_1 \phi_p \cos^2 \psi_c \quad y_f = r_1 - r_1 \phi_p \sin \psi_c \cos \psi_c \quad z_f = 0 \quad (4.4.9)$$

Equations (4.4.9) represent the line of action as a tangent to the base cylinder of radius  $r_{b1}$  that lies in plane  $z_f = 0$  and passes through point I (Fig. 4.4.1).

The pinion tooth surface may be represented in coordinate system  $S_1$  as the set of contact lines of surfaces  $\Sigma_p$  and  $\Sigma_1$ . Thus we obtain

$$[r_1] = [M_{lc}] [r_c^{(p)}] = [M_{lf}] [M_{fc}] [r_c^{(p)}] \quad (4.4.10)$$

$$f_1(u_p, \theta_p, \phi_p) = 0$$

here (Fig. 4.4.1)

$$[M_{lf}] = \begin{bmatrix} \cos \phi_p & \sin \phi_p & 0 & 0 \\ -\sin \phi_p & \cos \phi_p & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.4.11)$$

Equations (4.4.10), (4.4.3) and (4.3.8) yield

$$x_1 = u_p [\cos \theta_p \sin \alpha \cos(\alpha - \psi_c + \phi_p) - \cos \alpha \sin(\alpha - \psi_c + \phi_p)] \\ - u_p^* \sin(\psi_c - \phi_p) + r_1 (-\phi_p \cos \phi_p + \sin \phi_p)$$

$$y_1 = -u_p [\cos \theta_p \sin \alpha \sin(\alpha - \psi_c + \phi_p) + \cos \alpha \cos(\alpha - \psi_c + \phi_p)] \\ + u_p^* \cos(\psi_c - \phi_p) + r_1 (\phi_p \sin \phi_p + \cos \phi_p)$$

$$z_1 = u_p \sin \phi_p \sin \alpha$$

$$f_1(u_p, \theta_p, \phi_p) = r_1 \phi_p [-\cos \alpha \cos \theta_p \sin(\alpha - \psi_c) + \sin \alpha \cos(\alpha - \psi_c)] \\ - u_p \cos \theta_p + u_p^* (\cos \theta_p \cos^2 \alpha + \sin^2 \alpha) = 0 \quad (4.4.12)$$

Equations (4.4.12) are equivalent to the representation of the pinion surface by the vector function  $\xi_1(u_p, \theta_p, \phi_p)$  and the equation of meshing  $f_1(u_p, \theta_p, \phi_p) = 0$ . Taking it into account that  $f_1(u_p, \theta_p, \phi_p) = 0$  is linear with respect to the parameter  $u_p$ , it is easy to eliminate  $u_p$  and represent the pinion surface in two parametric form, with the parameters,  $\theta_p$  and  $\phi_p$ .

The intersection of the surface by the plane  $z_1 = 0$  represents a regular involute curve. Equations of this curve may be derived from the surface Eq. (4.4.12), taking it into account that

$$\theta_p = 0 \quad \text{and} \quad u_p - u_p^* = r_1 \phi_p \sin \psi_c \quad (4.4.13)$$

Thus, we have

$$x_1 = r_1 [-\phi_p \cos \psi_c \cos(\phi_p - \psi_c) + \sin \phi_p] \quad (4.4.14)$$

$$y_1 = r_1 [\phi_p \cos \psi_c \sin(\phi_p - \psi_c) + \cos \phi_p]$$

Cutting the pinion tooth surface by a plane  $z_1 = \text{const}$ , we obtain the cross-section of the pinion tooth surface that is deviated from a regular involute curve (Fig. 4.4.2).

It will be shown later that the kinematic errors caused by the gear misalignment can be of two types that are shown in Fig. 1.2a and 1.2b. Kinematic errors shown in Fig. 1.2a correspond to the case when the transformation of rotation is interrupted and the gear tooth surfaces are out of contact. To avoid the appearance of such kinematic errors, a surface of revolution that slightly deviates from the cone surface is actually used. The determination of curvatures of the surface of revolution is a subject of optimization. Since a surface of revolution is used instead of a cone surface, the crowned pinion surface deviates from a regular involute surface in all cross-sections including  $z_1 = 0$ , the middle cross-section of the tooth surface.

#### 4.5. Conditions of Pinion Without Undercutting

##### General Approach

The problem of undercutting of the pinion tooth surface by crowning is related with the appearance on the pinion tooth surface of singular points. It is known from differential geometry that the surface point is singular if the surface normal is equal to zero at such a point. Litvin, proposed a method (Litvin, 1987) to determine a line on the tool surface which will generate singular points on the surface that is generated by the tool. This line designated by L (Fig. 4.5.1) must be out of the working part of the tool surface to avoid undercutting of the pinion by crowning.

The limiting L of the tool surface is determined by the following equations

$$z_C^{(P)} = z_C^{(P)}(u_P, \theta_P) \quad (4.5.1)$$

$$f_1(u_P, \theta_P, \phi_P) = 0 \quad (4.5.2)$$

$$F(u_P, \theta_P, \phi_P) = 0 \quad (4.5.3)$$

Vector Eq. (4.5.1) represents the tool surface, Eq. (4.5.2) is the equation of meshing (see Eq. (4.4.3)) and Eq. (4.5.3) follows from the requirement that (Litvin, 1987)

$$\begin{array}{c|c} \begin{array}{ccc} \frac{\partial x_C^{(P)}}{\partial u_P} & \frac{\partial x_C^{(P)}}{\partial \theta_P} & -v_{cx}^{(C1)} \\ \frac{\partial y_C^{(P)}}{\partial u_P} & \frac{\partial y_C^{(P)}}{\partial \theta_P} & -v_{cy}^{(C1)} \\ \frac{\partial z_C^{(P)}}{\partial u_P} & \frac{\partial z_C^{(P)}}{\partial \theta_P} & -v_{cz}^{(C1)} \\ \frac{\partial f_1}{\partial u_P} & \frac{\partial f_1}{\partial \theta_P} & -\frac{\partial f_1}{\partial \phi_P} \cdot \frac{d\phi_P}{dt} \end{array} & = \begin{array}{ccc} \frac{\partial x_C^{(P)}}{\partial u_P} & \frac{\partial x_C^{(P)}}{\partial \theta_P} & -v_{cx}^{(C1)} \\ \frac{\partial z_C^{(P)}}{\partial u_P} & \frac{\partial z_C^{(P)}}{\partial \theta_P} & -v_{cz}^{(C1)} \\ \frac{\partial f_1}{\partial u_P} & \frac{\partial f_1}{\partial \theta_P} & -\frac{\partial f_1}{\partial \phi_P} \cdot \frac{d\phi_P}{dt} \end{array} \\ \hline \begin{array}{ccc} \frac{\partial x_C^{(P)}}{\partial u_P} & \frac{\partial y_C^{(P)}}{\partial \theta_P} & -v_{cy}^{(C1)} \\ \frac{\partial z_C^{(P)}}{\partial u_P} & \frac{\partial z_C^{(P)}}{\partial \theta_P} & -v_{cz}^{(C1)} \\ \frac{\partial f_1}{\partial u_P} & \frac{\partial f_1}{\partial \theta_P} & -\frac{\partial f_1}{\partial \phi_P} \cdot \frac{d\phi_P}{dt} \end{array} & = 0 \end{array} \quad (4.5.4)$$

### Derivation of Equation

Relative velocity  $\tilde{v}_c^{(C1)}$  is to be represented in coordinate system  $s_c$  as:

$$\tilde{v}_c^{(C1)} = \tilde{v}_c^{(C)} - \tilde{v}_c^{(1)} \quad (4.5.5)$$

where  $\tilde{v}_c^{(C)}$  is the velocity of the cutter and  $\tilde{v}_c^{(1)}$  is the velocity of the pinion at generating point. Using the sketch in Fig. 4.2.1, we get

$$\tilde{v}_c^{(C)} = \begin{bmatrix} \frac{ds}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -r_1 \omega_p \\ 0 \\ 0 \end{bmatrix} \quad (4.5.6)$$

$$\tilde{v}_c^{(1)} = \omega_p \times r_c + \overline{o_c o_1} \times \omega_p \quad (4.5.7)$$

Deriving Eq. (4.5.7) we substituted the sliding vector  $\omega_p$  that passes through  $o_1$  by the equal vector that passes through  $o_c$  and the vector-moment  $\overline{o_c o_1} \times \omega_p$ .

Here:

$$\omega_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \quad \overline{o_c o_1} = \begin{bmatrix} r_1 \phi_p \\ -r_1 \\ 0 \end{bmatrix} \quad (4.5.8)$$

Equations from (4.5.6) to (4.5.8) yield

$$\dot{\underline{v}}_c^{(C1)} = \begin{bmatrix} y_c^{(P)} \\ -x_c^{(P)} + r_1 \phi_P \\ 0 \end{bmatrix} \omega \quad (4.5.9)$$

where  $x_c^{(P)}$  and  $y_c^{(P)}$  are represented by equations (4.3.8). Equations (4.3.2), (4.3.3), (4.4.3) and (4.5.4) yield

$$F_1(u_p, \theta_p) = m_1 m_2 q - (m_2 + m_1 m_4) qp - m_3 m_5 q + m_4 p^2 = 0 \quad (4.5.10)$$

Here

$$q = \frac{u_p \cos \alpha}{d}$$

$$p = \frac{u_p^* \cos \psi_c}{d/\cos \alpha}$$

$$m_1 = \sin \alpha \sin(\alpha - \psi_c) \cos \theta_p + \cos \alpha \cos(\alpha - \psi_c)$$

$$m_2 = \sin \alpha \cos(\alpha - \psi_c) - \cos \alpha \sin(\alpha - \psi_c) \cos^3 \theta_p$$

$$m_3 = [\sin \alpha \cos(\alpha - \psi_c) - \cos \alpha \sin(\alpha - \psi_c) \cos \theta_p]^3$$

$$m_4 = \sin \alpha \cos \alpha \sin^2 \theta_p$$

$$m_5 = \frac{r_1 \cos \alpha}{d}$$

Using Eq. (4.5.10) and taking it into account that  $\frac{\partial F_1}{\partial \theta_p} \neq 0$ , we can represent  $u_p$  as function ( $\theta_p$ ). Undercutting will be avoided if

$$u_p(\theta_p) > \frac{d}{\cos \alpha} \quad (4.5.11)$$

The analysis of Eq. (4.5.10) yields that inequality (4.5.11) is satisfied if the conditions of non-undercutting of the pinion generated by a regular rack cutter are satisfied.

#### **4.6. Principal Directions and Curvatures of Tooth Surfaces**

##### **Introduction to Pinion Principal Directions and Curvatures**

The direct determination of pinion principal directions and curvatures requires complicated derivations. A simplified approach for the solution to this problem has been proposed by Litvin, in 1969. The main idea is as follows:

Consider that the tool surface  $\Sigma_p$  generates surface  $\Sigma_1$  as the envelope of the family of surfaces  $\Sigma_p$ . Let  $L_{p1}$  be the instantaneous line of contact of  $\Sigma_p$  and  $\Sigma_1$  and  $M$  is a point of this line. We consider as given the principal curvatures and directions of  $\Sigma_p$  at any point and also the parameters of motion of surfaces  $\Sigma_p$  and  $\Sigma_1$  being in mesh. Then the principal curvatures and directions of the generated surface  $\Sigma_1$  can be expressed in terms of principal curvatures and directions of the generating surface  $\Sigma_p$  by using the equations that have been derived by Litvin, 1969.

##### **Principal Curvatures and Directions of $\Sigma_p$**

The tool surface is a cone surface and its principal directions coincide with the direction of the cone generatrix and the direction that is perpendicular to the cone generatrix. The Rodrigues' formula (Eq. 4.6.1) can be used to find the principal curvatures and

directions.

$$\kappa_{I,\Pi} \cdot \tilde{v}_r = -\dot{\tilde{n}}_r \quad (4.6.1)$$

Here:  $\kappa_{I,\Pi}$  are the principal curvatures of surface.  $\tilde{v}_r$  is the velocity of a point that moves over a surface and  $\dot{\tilde{n}}_r$  is the derivative of the surface unit normal  $\tilde{n}$ , when  $\tilde{n}$  changes its direction due to the motion over the surface.

Equation (4.6.1) yields the following expressions for the principal curvatures and directions (see Eq. (4.3.1) for the tool cone surface  $\Sigma_P$  represented in coordinate system  $S_a$ )

$$\tilde{e}_I^{(P)} = \frac{\partial \tilde{r}_a}{\partial \theta_P} \div \left| \frac{\partial \tilde{r}_a}{\partial \theta_P} \right| = \begin{bmatrix} -\sin \theta_P \\ 0 \\ \cos \theta_P \end{bmatrix} \quad (4.6.2)$$

$$\kappa_I^{(P)} = -\frac{1}{u_P \tan \alpha} \quad (4.6.3)$$

$$\tilde{e}_{II}^{(P)} = \frac{\partial \tilde{r}_P}{\partial u_P} \div \left| \frac{\partial \tilde{r}_P}{\partial u_P} \right| = \begin{bmatrix} \cos \theta_P \sin \alpha \\ -\cos \alpha \\ \sin \theta_P \sin \alpha \end{bmatrix} \quad (4.6.4)$$

$$\kappa_{II}^{(P)} = 0 \quad (4.6.5)$$

The negative sign for  $\kappa_I$  indicates that the curvature center is located on the negative direction of the surface normal.

The principal curvatures are invariants with respect to the used

coordinate system. The unit vectors of the principal directions,  $\hat{e}_I^{(P)}$  and  $\hat{e}_{II}^{(P)}$ , may be represented in coordinate system  $S_f$  by using the following matrix equation

$$[\hat{e}_{I,II}^{(P)}]_f = [L_{fc}] [L_{ca}] [\hat{e}_{I,II}^{(P)}]_a \quad (4.6.6)$$

Here:  $[\hat{e}_{I,II}^{(P)}]_a$  and  $[\hat{e}_{I,II}^{(P)}]_f$  are vectors represented in  $S_a$  and  $S_f$ , respectively; matrix  $[L_{ca}]$  is represented by Eq. (4.3.10); matrix  $[L_{fc}]$  is the 3x3 unitary matrix (see matrix Eq. (4.4.8)).

Equation (4.6.6) yields

$$[\hat{e}_I^{(P)}]_f = \begin{bmatrix} -\sin\theta_p \cos(\alpha - \psi_c) \\ \sin\theta_p \sin(\alpha - \psi_c) \\ \cos\theta_p \end{bmatrix} \quad (4.6.7)$$

$$[\hat{e}_{II}^{(P)}]_f = \begin{bmatrix} \cos\theta_p \sin\alpha \cos(\alpha - \psi_c) - \cos\alpha \sin(\alpha - \psi_c) \\ -\cos\theta_p \sin\alpha \sin(\alpha - \psi_c) - \cos\alpha \cos(\alpha - \psi_c) \\ \sin\theta_p \sin\alpha \end{bmatrix} \quad (4.6.8)$$

### Principal Curvatures and Directions of Pinion Tooth Surface

The determination of principal curvatures and directions for the pinion tooth surface is based on the following equations (see Litvin, 1987)

$$\tan \angle_\sigma^{(P1)} = - \frac{2b_{13}b_{23}}{b_{23}^2 - b_{13}^2 - (\kappa_I^{(P)} - \kappa_{II}^{(P)})b_{33}} \quad (4.6.9)$$

$$\kappa_{II}^{(1)} - \kappa_I^{(1)} = \frac{b_{23}^2 - b_{13}^2 - (\kappa_I^{(P)} - \kappa_{II}^{(P)})b_{33}}{b_{33}\cos 2\sigma^{(Pl)}} \quad (4.6.10)$$

$$\kappa_{II}^{(1)} + \kappa_I^{(1)} = \kappa_I^{(P)} + \kappa_{II}^{(P)} + \frac{b_{13}^2 + b_{23}^2}{b_{33}^2} \quad (4.6.11)$$

Here:  $\kappa_I^{(P)}$  and  $\kappa_{II}^{(P)}$  are the principal curvatures of the tool surface  $\Sigma_P$ ;  $\tilde{e}_I^{(P)}$  and  $\tilde{e}_{II}^{(P)}$  are the unit vectors of the principal directions on  $\Sigma_P$  (Fig. 4.6.1);  $\kappa_I^{(1)}$  and  $\kappa_{II}^{(1)}$ ,  $\tilde{e}_I^{(1)}$  and  $\tilde{e}_{II}^{(1)}$  are the principal curvatures and directions on the pinion tooth surface  $\Sigma_1$ ; and  $\sigma^{(Pl)}$  is measured counterclockwise from  $\tilde{e}_I^{(P)}$  to  $\tilde{e}_I^{(1)}$  (Fig. 4.6.1). The coefficients  $b_{13}$ ,  $b_{23}$  and  $b_{33}$  have been derived from equations represented in [Litvin 1987] but modified for the case when a rack cutter generates a gear. The expressions for  $b_{13}$ ,  $b_{23}$  and  $b_{33}$  are as follows

$$\begin{vmatrix} b_{13} \\ b_{23} \end{vmatrix} = \begin{vmatrix} \tilde{e}_{II}^{(P)} \cdot \tilde{\omega}_P \\ -\tilde{e}_I^{(P)} \cdot \tilde{\omega}_P \end{vmatrix} - \begin{vmatrix} -\kappa_I^{(P)} & 0 \\ 0 & -\kappa_{II}^{(P)} \end{vmatrix} \begin{vmatrix} v_I^{(Pl)} \\ v_{II}^{(Pl)} \end{vmatrix} \quad (4.6.12)$$

$$b_{33} = [\tilde{n}_P \tilde{v}_{tr}^{(P)}] + [\tilde{n}_P \tilde{v}^{(Pl)}]$$

$$-\kappa_I^{(P)} (v_I^{(Pl)})^2 - \kappa_{II}^{(P)} (v_{II}^{(Pl)})^2 - \frac{m_1^2}{m_{1P}} m_{1P} [\tilde{n}_P \tilde{k}_f] \quad (4.6.13)$$

$$\text{where } m_{1P} = \frac{v_1}{s} = \frac{r_1 \omega_P}{s}$$

The vectors that are used in Eqs. (4.6.12) and (4.6.13) are represented in coordinate system  $S_f$  (Fig. 4.2.1);  $i_f$ ,  $j_f$  and  $k_f$  are

the unit vectors of coordinate axes of system  $S_f$ . The expressions of these vectors are as follows

$$\omega_p = \omega k_{\tilde{r}_f} \quad (4.6.14)$$

is the angular velocity of the pinion being in mesh with the rack cutter

$$v_{tr}^{(P)} = -\omega r_1 i_{\tilde{r}_f} \quad (4.6.15)$$

is the transfer velocity of a point of the rack cutter that performs translational motion;  $r_1$  is the radius of the pinion centrode

$$v_{tr}^{(1)} = \omega_p \times r_{\tilde{r}}^{(P)} = \begin{vmatrix} i & j & k \\ 0 & 0 & \omega \\ x_f & y_f & z_f \end{vmatrix} = \omega \begin{bmatrix} -y_f \\ x_f \\ 0 \end{bmatrix} \quad (4.6.16)$$

is the transfer velocity of the pinion. The term, "transfer velocity" means that the velocity of a point that is rigidly connected to the tooth surface is considered. Such a point performs the motion with the tooth surface

$$v_{tr}^{(P1)} = v_{tr}^{(P)} - v_{tr}^{(1)} = \omega \begin{bmatrix} y_f - r_1 \\ -x_f \\ 0 \end{bmatrix} \quad (4.6.17)$$

is the "sliding" velocity--the velocity of a point of the rack cutter

with respect to the same point of the pinion. Vector  $\tilde{v}^{(P1)}$  can be represented in terms of components  $x_c$  and  $y_c$  using matrix Eq. (4.4.8) that describes the coordinate transformation from  $S_c$  to  $S_f$ . Then we obtain

$$x_f = x_c - r_1 \phi_p \quad y_f = y_c + r_1 \quad (4.6.18)$$

Equations (4.6.17) and (4.6.18) yield

$$\tilde{v}^{(P1)} = \omega \begin{bmatrix} y_c \\ -x_c + r_1 \phi_p \\ 0 \end{bmatrix} \quad (4.6.19)$$

The principal curvatures of the cone that is rigidly connected to the rack cutter are represented as

$$\kappa_I^{(P)} = -\frac{1}{u_p \tan \alpha} \quad \kappa_{II}^{(P)} = 0 \quad (4.6.20)$$

The unit vectors of principal directions are designated by  $\tilde{e}_I^{(P)}$  and  $\tilde{e}_{II}^{(P)}$ . Components  $v_I^{(P1)}$  and  $v_{II}^{(P1)}$  are represented as follows

$$v_I^{(P1)} = \tilde{v}^{(P1)} \cdot \tilde{e}_I^{(P)} \quad v_{II}^{(P1)} = \tilde{v}^{(P1)} \cdot \tilde{e}_{II}^{(P)} \quad (4.6.21)$$

After substitution we obtain

$$v_{I_3} = \frac{\omega \sin \phi_p}{\sin \alpha} [1 - \frac{u_p^*}{u_p} \cos^2 \alpha + \frac{r_1 \phi_p}{u_p} \cos \alpha \sin(\alpha - \psi_c)] \quad (4.6.22)$$

$$b_{23} = -\omega \cos \theta_p \quad (4.6.23)$$

$$\omega_{33} = \omega^2 (\kappa_1 + \kappa_2 + \kappa_3 + \kappa_4) \quad (4.6.24)$$

where

$$\kappa_1 = u_p \sin^2 \theta_p \cos \alpha \sin \alpha - u_p^* \sin \alpha \cos \alpha (1 - \cos \theta_p) \quad (4.6.25)$$

$$\kappa_2 = r_1 \phi_p [\cos \theta_p \cos \alpha \cos(\alpha - \psi_c) + \sin \alpha \sin(\alpha - \psi_c)] \quad (4.6.26)$$

$$\kappa_3 = r_1 [\cos \theta_p \cos \alpha \sin(\alpha - \psi_c) - \sin \alpha \cos(\alpha - \psi_c)] \quad (4.6.27)$$

$$\kappa_4 = \frac{\sin^2 \theta_p}{u_p \tan \alpha} [(u_p^* - u_p) \cos \alpha - r_1 \phi_p \sin(\alpha - \psi_c)]^2 \quad (4.6.28)$$

For the contact point located in the middle section of the pinion we have  $\theta_p = 0$  and

$$b_{13} = 0 \quad b_{23} = \omega \quad b_{33} = \omega^2 [r_1 \phi_p \cos \psi_c - r_1 \sin \psi_c] \quad (4.6.29)$$

Then we obtain the following expressions for the pinion principal curvatures and directions

$$\sigma^{(P1)} = 0 \quad \kappa_I^{(1)} = \kappa_I^{(P)} = -\frac{1}{u \tan \psi_c}$$

$$\kappa_{II}^{(1)} = \kappa_{II}^{(P)} - \frac{1}{r_1 \sin \psi_c - r_1 \phi_p \cos \psi_c} = \frac{-1}{r_1 \sin \psi_c - r_1 \phi_p \cos \psi_c}$$

$$\eta = \eta^{(P)} \quad e_I^{(1)} = e_I^{(P)} \quad e_{II}^{(1)} = e_{II}^{(P)} \quad (4.6.30)$$

The principle curvature  $\kappa_{II}^{(1)}$  is the same as the principal curvature of a regular involute surface. However  $\kappa_I^{(1)}$  differs from zero because the pinion is generated by a cone but not a plane as in the case of a regular involute surface.

#### Gear Principal Curvatures and Directions

The determination of the gear principal curvatures is based on the relation between the principal curvatures for spur involute gears. Figure 4.6.2 shows that two mating involute shapes are in tangency at point M. Let us designate the principal curvature for the pinion by  $\kappa_I^{(1)}$  and  $\kappa_{II}^{(1)}$  where (see Eq. 4.6.29)

$$\kappa_I^{(1)} = -\frac{1}{u \tan \psi_c} \quad \kappa_{II}^{(1)} = -\frac{1}{r_1 \sin \psi_c - r_1 \phi_p \cos \psi_c} \quad (4.6.31)$$

The negative sign of  $\kappa_I^{(1)}$  and  $\kappa_{II}^{(1)}$  means that the radii of curvatures are opposite to the direction of the surface normal. The principal gear curvature  $\kappa_{II}^{(2)} = 0$ . The radii of curvature of spur involute gears are related as follows (Fig. 4.6.2) where  $\kappa_{II}^{(1)} = -\frac{1}{r_1 \sin \psi_c - r_1 \phi_p \cos \psi_c}$

$$\left| \frac{1}{\kappa_{II}^{(1)}} \right| + \left| \frac{1}{\kappa_{II}^{(2)}} \right| = KL = \frac{r_{p1} + r_{b2}}{\tan \psi_c} = \frac{r_1 + r_2}{\sin \psi_c} = \frac{N_1 + N_2}{2P \sin \psi_c} \quad (4.6.32)$$

Then we obtain

$$\frac{1}{\kappa_{\pi}^{(2)}} = \frac{r_1 + r_2}{\sin \psi_c} - \left| \frac{1}{\kappa_{II}^{(1)}} \right| = \frac{N_1 + N_2}{2p \sin \psi_c} - \left| \frac{1}{\kappa_{II}^{(1)}} \right| \quad (4.6.33)$$

The angle  $\sigma^{(21)}$  between the principal curvatures is equal to zero.

#### 4.7. Contact Ellipse and Bearing Contact

The dimensions and orientation of the instantaneous contact ellipse can be determined on the basis of equations that have been developed by Litvin (Litvin et al. 1982, and Litvin 1987). The input data for the computation is:  $\kappa_I^{(i)}$ ,  $\kappa_{II}^{(i)}$  ( $i=1,2$ ),  $\sigma^{(12)}$  and  $\epsilon$  where  $\kappa_I^{(i)}$  and  $\kappa_{II}^{(i)}$  are the principal curvatures,  $\sigma^{(12)}$  is the angle formed by the unit vectors of principal directions,  $e_I^{(1)}$  and  $e_I^{(2)}$  and  $\epsilon$  is the elastic deformation of the contacting surfaces at contact point. The bearing contact is formed by the set of instantaneous contact ellipses that move over the gear tooth surfaces in the process of meshing. In our case, when the crowned pinion is meshing with a gear without misalignment,  $e_I^{(1)} = e_I^{(2)}$  and  $\sigma^{(12)} = 0$  (Fig. 4.7.1). The axes of the contact ellipse are directed along the  $n$ -axis and  $\xi$ -axis, respectively. Generally the orientation of the contact ellipse is determined with the angle  $\psi_1$ . In the case where  $\sigma^{(12)} = 0$ ,  $\psi_1$  is equal to zero. The equations to be used for the computation are as follows

$$A = \frac{1}{4} [r_{\epsilon}^{(1)} - r_{\epsilon}^{(2)} - |g_1 - g_2|] \quad (4.7.1)$$

$$v = \frac{1}{4} [ \kappa_{\varepsilon}^{(1)} - \kappa_{\varepsilon}^{(2)} + |g_1 - g_2| ] \quad (4.7.2)$$

where

$$\kappa_{\varepsilon}^{(1)} = \kappa_I^{(1)} + \kappa_{II}^{(1)} \quad \kappa_{\varepsilon}^{(2)} = \kappa_I^{(2)} + \kappa_{II}^{(2)} \quad (4.7.3)$$

$$g_1 = \kappa_I^{(1)} - \kappa_{II}^{(1)} \quad g_2 = \kappa_I^{(2)} - \kappa_{II}^{(2)} \quad (4.7.4)$$

$$a^* = |\frac{\varepsilon}{A}|^{1/2} \quad b^* = |\frac{\varepsilon}{B}|^{1/2} \quad (4.7.5)$$

The equations for  $\kappa_I^{(i)}$  and  $\kappa_{II}^{(i)}$  ( $i=1,2,$ ) have been represented in Section 4.6. Figure 4.7.2 illustrates the bearing contact for spur gears with the crowned pinion tooth surface.

#### 4.8. Simulation of Meshing and Determination of Kinematical Errors

##### Coordinate Systems

The simulation of meshing is a part of the computer aided TCA program developed by the authors. The simulation of meshing is based on equations that provide the continuous tangency of contacting surfaces.

Henceforth we will use the following coordinate systems: (i)  $S_f$  that is rigidly connected to the frame and (ii) an auxiliary coordinate system  $S_n$  that is also rigidly connected to the frame, and (iii) systems  $S_1$  and  $S_2$  that are rigidly connected to the pinion and the gear, respectively. The origins and coordinates axes  $z_f$  and  $z_1$  of coordinate systems  $S_f$  and  $S_1$  coincide with each other (Fig. 4.8.1a).

Axis  $z_f$  is the axis of rotation of the pinion. Also, the origins and coordinate axes  $z_h$  and  $z_2$  coincide with each other and  $z_h$  is the axis of gear rotation (Fig. 4.8.1b). The errors of assembly of the gears are simulated with the orientation and location of coordinate system  $S_h$  with respect to  $S_f$ . Figure 4.8.2 shows the orientation

(a) The gear axes are crossed (Fig. 4.8.2a)

$$[M_{fh}] = \begin{bmatrix} -\cos\Delta\gamma & 0 & \sin\Delta\gamma & 0 \\ 0 & -1 & 0 & C \\ \sin\Delta\gamma & 0 & \cos\Delta\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.8.1)$$

(b) The gear axes are intersected (Fig. 4.8.2b)

$$[M_{fh}] = \begin{bmatrix} -1 & 0 & C & 0 \\ 0 & -\cos\Delta\delta & -\sin\Delta\delta & C \\ 0 & -\sin\Delta\delta & \cos\Delta\delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.8.2)$$

### Pinion Tooth Surface Equations

The pinion tooth surface has been represented in coordinate system  $S_1$  by Eq. (4.4.12). The coordinate transformation from  $S_1$  to  $S_f$  is represented as follows

$$[r_f^{(1)}] = [M_{f1}] [r_1] \quad (4.8.3)$$

Here

$$[M_{f1}] = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.8.4)$$

where  $\phi_1$  is the angle of rotation of the pinion being in mesh with the gear.

Equation (4.4.12) represent the pinion tooth surface in a three-parametric form in terms of parameters  $u_p$ ,  $\theta_p$  and  $\phi_p$  where  $\phi_p$  is the angle of rotation of the pinion being in mesh with the coned cutter. These parameters are related by the equation of meshing and only two of them are independent. It is easy to eliminate parameter  $u_p$  and represent the pinion tooth surface in  $S_1$  in the two-parametric form.

Thus:

$$\xi_1 = \xi_1(\phi_p, \theta_p) \quad (4.8.5)$$

where  $\phi_p$  and  $\theta_p$  serve as the surface parameters. The pinion tooth surface unit normal can be represented in coordinate system  $S_f$  as

$$[\mathbf{n}_f^{(1)}] = [L_{f1}] [\mathbf{n}_1] \quad (4.8.6)$$

The  $3 \times 3$  matrix  $[L_{f1}]$  can be developed from  $[M_{f1}]$  by eliminating the last row and the last column. The surface unit normal,  $\mathbf{n}_1$ , is represented in  $S_1$  by the following matrix equation

$$[\mathbf{n}_1] = [L_{1c}] [\mathbf{n}_c^{(P)}] = [L_{1f}] [L_{fc}] [\mathbf{n}_c^{(P)}] \quad (4.8.7)$$

Here:  $\tilde{n}_c^{(P)}$  is represented by Eqs. (4.3.11),  $[L_{fc}]$  is the 3x3 unitary matrix and  $[L_{lf}]$  is the 3 x 3 submatrix of matrix  $[M_{lf}]$  given by Eq. (4.4.11).

The final expression for  $\tilde{n}_f^{(1)}$  is as follows

$$[n_f^{(1)}] = [L_{f1}] [L_{lf}] [L_{fc}] [n_c^{(P)}] \quad (4.8.8)$$

Here:  $[L_{fc}]$  is the 3x3 unitary matrix  $[I]$ , but  $[L_{f1}] [L_{lf}] \neq [I]$  because the elements of these matrices are expressed in different terms -  $\phi_p$  and  $\phi_1$

#### Gear Tooth Surface Equations

The gear tooth surface is a regular involute tooth surface and may be represented in coordinate system  $S_2$  by equations that are similar to Eq. (4.4.14)

$$x_2 = r_2 [-\phi_G \cos \psi_C \cos(\phi_G - \psi_C) + \sin \phi_G]$$

$$y_2 = r_2 [\phi_G \cos \psi_C \sin(\phi_G - \psi_C) + \cos \phi_G]$$

$$z_2 = \lambda \quad (4.8.9)$$

where  $\phi_G$  is the angle of rotation of the gear being in mesh with the rack cutter. The gear tooth surface unit normal is represented by the equations

$$\tilde{n}_2 = \frac{\tilde{z}_2}{|\tilde{z}_2|} \quad \tilde{n}_2 = \frac{\partial \tilde{r}_2}{\partial \phi_G} \times \frac{\partial \tilde{r}_2}{\partial \ell} \quad (4.8.10)$$

Equations (4.8.9) and (4.8.10) yield

$$[n_2] = \begin{bmatrix} -\cos(\phi_G - \psi_C) \\ \sin(\phi_G - \psi_C) \\ 0 \end{bmatrix} \quad (4.8.11)$$

To represent the gear tooth surface and its unit normal in coordinate system  $S_h$  we use the following matrix equations (Fig. 4.8.2)

$$[r_h^{(2)}] = [M_{h2}] [r_2] \quad [n_h^{(2)}] = [L_{h2}] [n_2] \quad (4.8.12)$$

$$[M_{h2}] = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 & 0 \\ \sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.8.13)$$

where  $\phi_2$  is the angle of rotation of the gear being in mesh with the pinion. Equations (4.8.9), (4.8.11), (4.8.12) and (4.8.13) yield

$$\begin{aligned} x_n^{(2)} &= -r_2 \{ \phi_G \cos \psi_C \cos[(\phi_2 - \phi_G) + \psi_C] + \sin(\phi_2 - \phi_G) \} \\ y_n^{(2)} &= -r_2 \{ \phi_G \cos \psi_C \sin[(\phi_2 - \phi_G) + \psi_C] - \cos(\phi_2 - \phi_G) \} \\ z_n^{(2)} &= \lambda \end{aligned} \quad (4.8.14)$$

$$[n_h^{(2)}] = \begin{bmatrix} -\cos[(\phi_2 - \phi_G) + \psi_C] \\ -\sin[(\phi_2 - \phi_G) + \psi_C] \\ 0 \end{bmatrix} \quad ((4.8.15))$$

These equations with parameters  $\phi_2 = \phi_c = 0$ ,  $z_h^{(2)} = \lambda = 0$  determine point M with coordinates  $(0, r_2, 0)$  and the surface unit normal at M with components  $(-\cos\psi_c, -\sin\psi_c, 0)$  (Fig. 4.8.3).

To determine the gear tooth surface equations and the surface unit normal in coordinate system  $S_f$  we use the following matrix equations

$$[r_f^{(2)}] = [M_{fh}] [r_h^{(2)}] \quad [n_f^{(2)}] = [L_{fh}] [n_h^{(2)}] \quad (4.8.16)$$

Matrix  $[M_{fh}]$  (see Eqs. (4.8.1) and (4.8.2)) describes the coordinate transformation from  $S_h$  to  $S_f$  for the case where the gear is misaligned with respect to the pinion. Figure 4.8.4 illustrates the case where the gears are not misaligned, the shortest distance of gear axes  $C = r_1 + r_2$  and the tooth surfaces of the pinion and the gear contact each other at point M. The misalignment of the gear will not cause the surface contact at the tooth edge since the pinion tooth surface is crowned and deviates from a regular involute surface.

### Equation of Tangency

The contact of gear tooth surfaces is simulated in the developed TCA program by the following equations

$$\xi_f^{(1)}(\phi_p, \theta_p, \phi_1) = \xi_f^{(2)}(\phi_G, \ell, \phi_2) \quad (4.8.17)$$

$$\eta_f^{(1)}(\phi_p, \theta_p, \phi_1) = \eta_f^{(2)}(\phi_G, \phi_2) \quad (4.8.18)$$

At the contact point vector Eq. (4.8.17) provides the equality of

position vectors and vector Eq. (4.8.18) provides the equality of surface unit normals. Vector Eq. (4.8.17) provides three independent scalar equations that relate  $\phi_P, \theta_P, \phi_1, \phi_G, \lambda$  and  $\phi_2$ . However, vector Eq. (4.8.18) provides only two independent scalar equations since  $\tilde{n}_f^{(1)}$  and  $\tilde{n}_f^{(2)}$  are unit vectors and  $|\tilde{n}_f^{(1)}| = |\tilde{n}_f^{(2)}| = 1$ . Thus Eqs. (4.8.17) and (4.8.18) yield five independent scalar equations as follows

$$f_i(\phi_P, \theta_P, \phi_1, \phi_G, \lambda, \phi_2) = 0 \quad (i=1, 2, \dots, 5) \quad (4.8.19)$$

Equation system (4.8.19) is the expression in implicit form of functions of one variable, for instance of  $\phi_1$ . The Theorem of Implicit Function System Existence states:

- (i) Consider that a set of parameters

$$P^o = (\phi_P, \theta_P, \phi_1, \phi_G, \lambda, \phi_2) \quad (4.8.20)$$

satisfies Eq. (4.8.19) and that the Jacobian

$$J = \frac{D(f_1, f_2, f_3, f_4, f_5)}{D(\phi_P, \theta_P, \phi_1, \lambda, \phi_2)} \neq 0 \quad (4.8.21)$$

(the partial derivatives are taken at point  $P^o$ ).

- (ii) Then, equation system (4.8.19) can be solved in the neighborhood of point  $P^o$  by functions

$$\phi_2(\phi_1), \quad \phi_P(\phi_1), \quad \theta_P(\phi_1), \quad \phi_G(\phi_1), \quad \lambda(\phi_1) \in C \quad (4.8.22)$$

The designation  $C^1$  means that the functions have continuous derivatives at least of the first order. Knowing function  $\phi_2(\phi_1)$ , we can determine the function of kinematical errors

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2} \phi_1 \quad (4.3.23)$$

#### 4.9 Modification of Generating Surface $\Sigma_p$

##### Basic Relations

The proposed method of crowning provides conjugated gear tooth surfaces and a localized bearing contact. Using the TCA program it can be proven that the kinematical errors caused by the gear misalignment are on a very low level. However, the shape of the function of kinematic errors is unfavorable, similar to the shape that has been represented in Fig. 1.2a. This disadvantage can be avoided by the modification of generating surface  $\Sigma_p$ . The authors proposed to use a surface of revolution instead of a cone surface. Figure 4.9.1 shows the axial section of generating surface  $\Sigma_p$  that is an arc of the circle of radius  $\rho$ . Controlling the magnitude of  $\rho$ , we can provide the favorable shape of the function of kinematical errors (Fig. 1.2b) and also control the level of kinematical errors. The solution to this problem is based on the relation between the principal curvatures and directions for the contacting surfaces and the derivative

$m'_{21} = \frac{d}{d\phi_1} (m_{21}\phi_1)$ . This relation (Litvin, 1987) is represented as follows

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \quad (4.9.1)$$

Consider that the tooth surfaces of a crowned pinion and a gear are in contact at point M that coincides with the instantaneous center of rotation (Fig. 4.8.4). It is assumed that the gears are not misaligned. In this case the coefficients of determinant (4.9.1) are expressed as follows

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa_I^{(2)} & 0 \\ 0 & -\kappa_I^{(1)} + \kappa_I^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = \begin{bmatrix} -e_{II}^{(1)} \cdot \omega^{(12)} \\ e_I^{(1)} \cdot \omega^{(12)} \end{bmatrix}$$

$$b_{33} = -n [(\omega^{(1)} x_{V+r}^{(2)}) - (\omega^{(2)} x_{V+r}^{(1)})] + [\omega^{(1)}]^2 m'_{21} (n x k_2) \cdot (r_f^{(1)} - R_f) \quad (4.9.2)$$

where

$$m_{21} = \frac{d\phi_2}{d\phi_1}, \quad m'_{21} = \frac{dm_{21}}{d\phi_1} = \frac{d}{d\phi_1} \left( \frac{d\phi_2}{d\phi_1} \right)$$

Equations (4.3.11), (4.6.6), (4.6.9) with  $\theta_p = 0$  yield (see Fig. 3.4)

$$\tilde{n} = \begin{bmatrix} \cos \psi_C \\ \sin \psi_C \\ 0 \end{bmatrix}$$

$$\tilde{e}_I^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\tilde{e}_{II}^{(2)} = \begin{bmatrix} \cos \psi_C \\ -\sin \psi_C \\ 0 \end{bmatrix}$$

$$\omega^{(12)} = \omega^{(1)} - \omega^{(2)} = -\omega^{(1)} \left(1 + \frac{N_1}{N_2}\right) k_f$$

where  $N_1$  and  $N_2$  are the number of teeth of the pinion and gear

$$k_f = (r_1 + r_2) \lambda_f$$

where  $r_1$  and  $r_2$  are the radii of the pitch circles

$$\tilde{x}_f^{(1)} = r_1 j_f$$

$$\tilde{v}_{tr}^{(1)} = \omega^{(1)} \times \tilde{x}_f^{(1)} = \omega^{(1)} r_1 i_f$$

$$\tilde{v}_{tr}^{(2)} = \omega^{(2)} \times (\tilde{x}_f^{(1)} - \tilde{k}_f) = \omega^{(1)} r_1 \dot{z}_f \quad (4.9.3)$$

Substituting Eqs. (4.9.3) into Eq. (4.9.2), we obtain

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa_I^{(2)} & 0 \\ 0 & -\kappa_{II}^{(1)} + \kappa_{II}^{(2)} \end{bmatrix}$$

$$v_{13} = 0$$

$$\omega_{23} = -\omega^{(1)} \left(1 + \frac{N_1}{N_2}\right) \quad (4.9.4)$$

$$\omega_{33} = \omega^{(2)} r_1 \left(1 + \frac{N_1}{N_2}\right) \sin \psi_C + m'_{21} \frac{N_2}{N_1} \cos \psi_C$$

Equations (4.9.1) and (4.9.4) yield the following relation between the derivative  $m'_{21}$  and the principal curvatures

$$m'_{21} = \frac{\left(1 + \frac{N_1}{N_2}\right)^2 - \left(1 + \frac{N_1}{N_2}\right) \sin \psi_C r_1 [-\kappa_{II}^{(1)} + \kappa_{II}^{(2)}]}{r_1 \cos \psi_C N_1 \left[-\kappa_{II}^{(1)} + \kappa_{II}^{(2)}\right]} \quad (4.9.5)$$

Here (see Eqs. (4.6.31) and (4.6.32))

$$\kappa_{II}^{(1)} = \kappa_{II}^{(P)} - \frac{1}{r_1 \sin \psi_C} \quad \kappa_{II}^{(2)} = \frac{1}{r_2 \sin \psi_C} = \frac{N_1/N_2}{r_1 \sin \psi_C} \quad (4.9.6)$$

$$m'_{21} = \left(1 + \frac{N_1}{N_2}\right) \frac{N_1}{N_2} \tan \psi_C \frac{\kappa_{II}^{(P)}}{\left(1 + \frac{N_1}{N_2}\right) \frac{1}{r_1 \sin \psi_C} - \kappa_{II}^{(P)}} \quad (4.9.7)$$

here (Fig. 4.9.1)

$$\kappa_{II}^{(P)} = -\frac{1}{s}$$

where  $s$  is the radius of the arc circle that generates the surface of revolution.

There are two important particular cases for the application of Eq. (4.9.7) (i) The generating surface  $\Sigma_p$  is a cone. Considering that  $\rho \rightarrow \infty$  and the generating surface becomes a cone, we obtain that  $m_{21}' = 0$  if the gears are not misaligned. In this case, when misalignment exists, kinematic errors will occur as shown in Fig. 1.2a which are very undesirable. (ii) The generating surface  $\Sigma_p$  is a revolute surface,  $\rho$  is positive,  $\kappa_{II}^{(P)} < 0$  ( $|\kappa_{II}^{(P)}| = \frac{1}{\rho}$ ) and  $m_{21}' < 0$  ( $m_{21}'$  is taken at point M). Then the function of kinematic errors will be similar to that as shown in Fig. 1.2b even when the gears are not aligned. It will be shown below that by choosing an appropriate value for  $\rho$  it is possible to keep the kinematical errors of misaligned gears on a very low level.

### Surface of Revolution Equations

The surface of revolution is generated by an arc of circle of radius  $\rho$ . The arc parameters provide a common normal for the cone surface and the surface of revolution at their point of tangency N (Fig. 4.9.2a). The circular arc is expressed in the auxiliary coordinate system  $S_e$  as follows

$$\begin{aligned}
 x_e &= \rho[\cos(\alpha+\beta) - \cos\alpha] + u_p^* \sin\alpha \\
 &= u_p^* \sin\alpha - 2\rho \sin^2 \left(\frac{\alpha+\beta}{2}\right) \\
 z_e &= \rho[\sin(\alpha+\beta) - \sin\alpha] + \frac{b}{\cos\beta} \cos\alpha \\
 &= \frac{b}{\cos\beta} \cos\alpha + 2\rho \sin^2 \frac{\beta}{2} \cos\left(\alpha+\frac{\beta}{2}\right) \\
 z_e &= v
 \end{aligned} \tag{4.9.9}$$

The unit normal to the arc at point M is also the normal to the cone surface and surface revolution at this point and it can be expressed as follows

$$[n_e] = \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \\ 0 \end{bmatrix} \quad (4.9.10)$$

The surface of revolution is generated by rotation of the circular arc about the  $y_e$ -axis and may be represented in coordinate system  $S_a$  as follows

$$[r_a] = [L_{ae}] [r_e] \quad [n_a] = [L_{ae}] [n_e] \quad (4.9.11)$$

here (fig. 4.9.20)

$$[L_{ae}] = \begin{bmatrix} \cos\theta_p & 0 & \sin\theta_p \\ 0 & 1 & 0 \\ -\sin\theta_p & 0 & \cos\theta_p \end{bmatrix} \quad (4.9.12)$$

Then we obtain

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} [U_p^* \sin\alpha - 2\sin\frac{\beta}{2} \sin(\alpha + \frac{\beta}{2})] \cos\theta_p \\ \cos\frac{\beta}{2} \cos\alpha + 2\sin\frac{\beta}{2} \cos(\alpha + \frac{\beta}{2}) \\ [U_p^* \sin\alpha - 2\sin\frac{\beta}{2} \sin(\alpha + \frac{\beta}{2})] \sin\theta_p \end{bmatrix} \quad (4.9.13)$$

$$[r_a] = \begin{bmatrix} \cos\theta_p \cos(\alpha + \beta) \\ \sin(\alpha + \beta) \\ \sin\theta_p \cos(\alpha + \beta) \end{bmatrix} \quad (4.9.14)$$

The installment of the generating surface in coordinate system  $S_C$  is the same as it was described in Section 4.3. The generating surface and its unit normal are represented in  $S_C$  as follows

$$[r_C^{(P)}] = [M_{Ca}] [r_a] = \quad (4.9.15)$$

$$\left[ \begin{array}{l} 2\rho \sin \frac{\beta}{2} [\sin(\alpha - \psi_C) \cos(\alpha + \frac{\beta}{2}) - \cos \theta_P \cos(\alpha - \psi_C) \sin(\alpha + \frac{\beta}{2})] \\ + U_P^* \sin \alpha \cos(\alpha - \psi_C) (\cos \theta_P - 1) \\ \\ 2\rho \sin \frac{\beta}{2} [\cos(\alpha - \psi_C) \cos(\alpha + \frac{\beta}{2}) + \cos \theta_P \sin(\alpha - \psi_C) \sin(\alpha + \frac{\beta}{2})] \\ - U_P^* \sin \alpha \sin(\alpha - \psi_C) (\cos \theta_P - 1) \\ \\ [U_P^* \sin \alpha - 2\rho \sin \frac{\beta}{2} \sin(\alpha + \frac{\beta}{2})] \sin \theta_P \end{array} \right]$$

$$[n_C^{(P)}] = [M_{Ca}] [n_a] = \quad (4.9.16)$$

$$\left[ \begin{array}{l} \cos \theta_P \cos(\alpha + \beta) \cos(\alpha - \psi_C) + \sin(\alpha + \beta) \sin(\alpha - \psi_C) \\ - \cos \theta_P \cos(\alpha + \beta) \sin(\alpha - \psi_C) + \sin(\alpha + \beta) \cos(\alpha - \psi_C) \\ \sin \theta_P \cos(\alpha + \beta) \end{array} \right]$$

### Pinion Surface

The pinion surface is represented in coordinate system  $S_1$  as follows

$$[r_1] = [H_{1f} \ L_{fc}] [r_c] \quad f(\beta_1, \theta_p, \phi_p) = 0 \quad (4.9.17)$$

here  $f(\beta, \theta_p, \phi_p) = 0$  is the equation of meshing (see Section 4.4) represented as follows

$$\begin{aligned} f(\beta, \theta_p, \phi_p) &= r_1 \sin \beta \cos \theta_p - u_p^* \sin \alpha \sin(\alpha + \beta)(\cos \theta_p - 1) \\ &\quad - r_1 \phi_p \sin(\alpha + \beta) \cos(\alpha - \psi_c) - \cos \theta_p \cos(\alpha + \beta) \sin(\alpha - \psi_c) = 0 \end{aligned} \quad (4.9.18)$$

### TCA Program

Following the procedure of derivations described in Section 4.8, we can now develop the TCA program for the case where the generating surface  $\Sigma_p$  is a surface of revolution. Using this program, we can determine the kinematical errors caused by the gear.

### Example

$$r_n = 10 \quad N_1 = 20, \quad N_2 = 40 \quad C = 1.01(r_1 + r_2) \quad \Delta\gamma = 5^\circ$$

$$\psi_c = 20^\circ \quad \alpha = 30^\circ \quad \rho = 500 \text{ in} \quad d = 0.176 \text{ in}$$

The results of computation of kinematical errors are shown in Fig. 4.9.3. It was found that the function of kinematical errors has two distinct shape, and the maximal value is 0.16 arc sec.

### **Conclusion**

The authors have developed: (i) The basic principles for the optimal geometry of crowned spur gears; (ii) New methods for generation of crowned pinion tooth surface; (iii) Tooth contact analysis programs for the simulation of meshing and bearing contact.

The features of the optimal geometry of the crowned spur gears are as follows: (i) the gear is provided with a regular involute surface; (ii) the pinion tooth surface is deviated from a regular involute surface to provide a localized bearing contact and reduce the sensitivity of the gears to their misalignment; (iii) the function of transmission errors of the misaligned gears is of a parabolic type with the small level of maximal errors, less than 5 arc seconds.

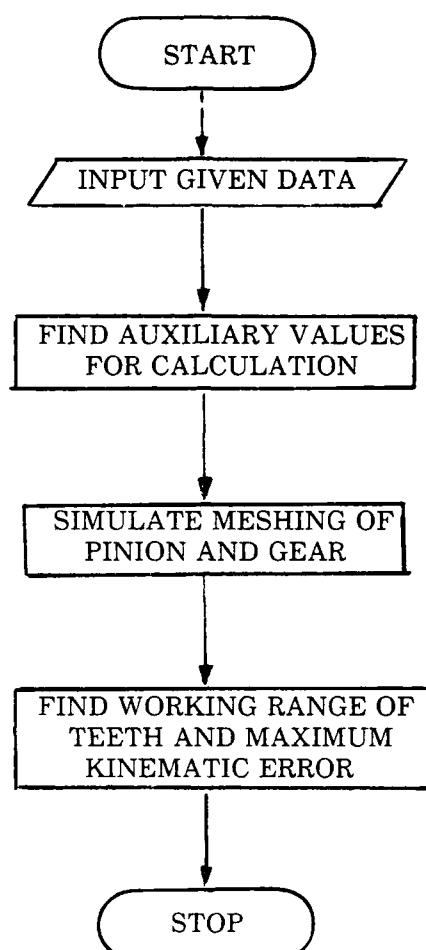
The new methods for generation of the crowned pinion surface need simple tools that have to be provided with a generating plane or a surface of revolution that slightly deviates from a cone surface. The developed TCA programs provide the information on the bearing contact and the transmission errors for the misaligned gears.

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FLOWCHART FOR PROGRAM I



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C... ****
C... *
C... *
C... * PROGRAM I
C... * KINEMATIC ERROR OF A REGULAR INVOLUTE GEAR MESHING *
C... * WITH A PINION CROWNED BASED ON PREDESIGNED *
C... * KINEMATIC ERROR FUNCTION
C... *
C... * AUTHORS: FAYDOR LITVIN
C... * JIAO ZHANG
C... *
C... ****
C
C PURPOSE
C
C THIS PROGRAM IS USED TO CALCULATE THE KINEMATIC ERROR OF A CROWNED
C PINION MESHING WITH A REGULAR INVOLUTE GEAR. THE PINION IS
C CROWNED BASED ON PREDESIGNED KINEMATIC ERROR FUNCTION.
C
C THIS PROGRAM IS WRITTEN IN FORTRAN77. IT CAN BEE COMPILED BY V
C COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
C IMPLICIT REAL*8(A-H,O-Z)
C DIMENSION Z(99),ERR(99),ERROR(99)
C COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
C EPSI,DELTA,NC,NE,NDIM
C COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,C,
C S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R,CONST(9)
C
C DEFINE PARAMETERS USED BY PROGRAMS
C
C (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C DEVICES
C     IN=5
C     LP=6
C (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
C     NDBUG=2
C (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C     EQUATIONS;
C     EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C     IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C     DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C     NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVRGENT
C     OR LESS ACCURATE
C     NC=100
C     DELTA=1.D-3
C     EPSI=1.D-12
C (4) OTHER PARAMETERS(DON'T CHANGE)
C     NDIM=10
C     NE=5
C     DR=DATAN(1.D0)/45.D0
C
C DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C     RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C     RKSI=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C     COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)

```

```

C      R=RADIUS OF ROTATION AXIS FOR GENERATING PINION SURFACE
PN=10.D0
N1=20
RMPG=2.D0
RKS1=20.D0*DR
HDC=1.D0/PN
COE=1.01D0
R=3.5D2
C      (2) PRE-DESIGNED FUNCTION: INPUT CONSTANTS HERE AND INPUT FUNCTION
C          IN SUBROUTINE FUNC
C          MATRIX CONST(J) IS USED FOR CONSTANTS OF FUNCTION
CONST(1)=2.D0*3.929751681D-4
C      (3) MISALIGNMENT: NMIS=ID NO. (1=CROSSING AXES, 2=INTERSECTING AXES);
C          NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C          (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
NMIS=2
NG=2
GAMMAI=5.D0
C      (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
FEEI=1.0D0*DR
C
C      DESCRIPTION OF OUTPUT PARAMETERS
C
C      FEE1=ROTATION ANGLE OF PINION
C      FEE2=ROTATION ANGLE OF GEAR
C      RP=RADIUS OF PINION CONTACT POINT
C      RG=RADIUS OF GEAR CONTACT POINT
C
C      FIND AUXILIARY VALUES FOR CALCULATION
RP=FLOAT(N1)/2./PN
RG=RP*RMPG
C=(RP+RG)*COE
CK=DCOS(RKS1)
SK=DSIN(RKS1)
NCOEF=360.D0*DR/FEEI/FLOAT(N1)+0.3D0
N=NCOEF*2+1
NT=N-NCOEF
FII=360.D0/RMPG/FLOAT(N1)
C      DEFINE MATRIX DESCIBEING MISALIGNMENT AND OUTPUT ASSEMBLING CONDITION
DO 5 I=1,3
DO 5 J=1,3
S(I,J)=0.D0
IF (I.EQ.J) S(I,J)=1.D0
5 CONTINUE
DO 15 LL=1,NG
GAMMA=GAMMAI*FLOAT(LL-1)/60.D0*DR
CG=DCOS(GAMMA)
SG=DSIN(GAMMA)
GAMMA=GAMMA/DR*60.D0
IF (NMIS.EQ.1) THEN
WRITE (LP,10) COE,GAMMA
10 FORMAT (1H1///,1X,'C=',F4.2,'*(RP+RG);      CROSSING ANGLE=',
        +         F5.1,'(M)')
S(1,1)=CG
S(1,3)=-SG
S(3,1)=SG
S(3,3)=CG
ELSE

```

```

      WRITE (LP,20) COE,GAMMA
20 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG);      INTERSECTING ANGLE=',
+          F5.1,'(M)')
      S(2,2)=CG
      S(2,3)=-SG
      S(3,2)=SG
      S(3,3)=CG
      END IF
C  SIMULATING MESHING OF PINION AND GEAR(L=1 FOR FINDING INITIAL
C  ROTATION CORRESPONDING TO 0 PINION ROTATION) AND OUTPUT RESULTS
      DO 25 L=1,2
      DO 35 I=1,4
35 X(I)=0.D0
      DO 45 I=1,N
      X(7)=FEEI*FLOAT(I-(N+1)/2)
      IF (L.EQ.1) X(7)=0.D0
      X(5)=DARSIN(RP*X(7)*SK/R)
      X(2)=X(7)/RMPG
      SF=DSIN(X(7))
      CF=DCOS(X(7))
      CALL NONLIN
      X(8)=X(2)+X(3)
      IF (L.EQ.1) THEN
      XIN=X(8)
      WRITE (LP,30)
30 FORMAT (///8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
+         'RP',13X,'RG')
      GO TO 25
      END IF
      X(8)=X(8)-XIN
      X(7)=X(7)/DR
      X(8)=X(8)/DR
      X(9)=(X(8)-X(7))/RMPG*3600.D0
      Z(I)=X(8)
      ERR(I)=X(9)
      WRITE (LP,40) (X(J),J=7,11)
40 FORMAT (1X,5F15.7,F15.7)
45 CONTINUE
      WRITE (LP,50)
50 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:/')
      DO 55 I=1,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE=Z(KK)-Z(I)
      ERROR(I)=(ANGLE-FII)*3600.D0
      WRITE (LP,60) X(7),X(8),ANGLE
60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
55 CONTINUE
      DO 65 I=1,NT
      ATEMP2=ERROR(I)
      IF (I.NE.1) THEN
      IF (ATEMP1.GT.0.D0.AND.ATEMP2.LE.0.D0) GOTO 75
      END IF
      ATEMP1=ATEMP2
65 CONTINUE
      WRITE (LP,70)
70 FORMAT (//1X,'MESHING IS DISCONTINOUS')

```

```

      GOTO 25
75 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.DO
      EMIN=0.DO
      DO 85 J=1,NCOEF
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
      IF (ET.GT.EMAX) EMAX=ET
85 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,80) Z(I),Z(KK),ET
80 FORMAT (//IX,'WORKING RANGE FOR THE GEAR TOOTH: ',F7.2,'----',
      +          F7.2/IX,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,' (S)')
25 CONTINUE
15 CONTINUE
      WRITE (LP,90)
90 FORMAT(1H1)
      STOP
      END

C      SUBROUTINE FUNC
C
C THIS SUBROUTINE IS USED TO GENERATE FIVE FUNCTIONS, THAT IS, AT
C CONTACT POINT, POSITION VECTOR AND NORMAL OF PINION AND GEAR
C MUST COINCIDE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
      +          EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,C,
      +          S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R,CONST(9)
C INPUT PRE-DESIGN ERROR FUNCTION EF AND ITS DIFFEVERTIVE DEF,
C RECALLING X(5) IS FEEP
      EF=-CONST(1)*X(5)*X(5)/2.DO
      DEF=-CONST(1)*X(5)
      CT=DCOS(X(4))

C
      ST=DSIN(X(4))
      CFEE=DCOS(X(3))
      SFEE=DSIN(X(3))
      CFEK=DCOS(X(3)+RKSI)
      SFEK=DSIN(X(3)+RKSI)
      GF=DARCOS((1.D0+DEF*RMPG/(1.D0+RMPG))*CK)
      CFGK=DCOS(X(5)+GF-RKSI)
      SFGK=DSIN(X(5)+GF-RKSI)
      CFP=DCOS(X(5))
      SFP=DSIN(X(5))
      CFPG=DCOS(X(5)+GF)
      SFPG=DSIN(X(5)+GF)
      T3=X(5)/RMPG+EF+RKSI-GF
      X11=-(RP+RG)*SFP+RG*SFGK+RG*T3*CK*CFPG
      Y11=(RP+RG)*CFP+RG*CFGK+RG*T3*CK*SFPG
      XP=X11*CT+R*(CT-1.D0)
      YP=Y11
      ZP=X11*ST+R*ST
      PNXP=CFPG*CT
      PNYP=SFPG

```

```

RNZP=CF*G*ST
XG=RG*S^EE+RG*X(2)*CK*CFEK
YG=-RG*CFEE+RG*X(2)*CK*SFEK
ZG=X(1)
RNXG=CFEK
RNYG=SFEK
RNZG=0.D0
Y(1)=CF*XP+SF*YP-S(1,1)*XG-S(1,2)*YG-S(1,3)*ZG
Y(2)=CF*YP-SF*XP-S(2,1)*XG-S(2,2)*YG-S(2,3)*ZG-C
Y(3)=ZP-S(3,1)*XG-S(3,2)*YG-S(3,3)*ZG
Y(4)=CF*RNYP-SF*RNXP-S(2,1)*RNXG-S(2,2)*RNYG-S(2,3)*RNZG
Y(5)=RNZP-S(3,1)*RNXG-S(3,2)*RNYG-S(3,3)*RNZG
X(10)=DSQRT(XP*XP+YP*YP)
X(11)=DSQRT(XG*XG+YG*YG)
C      WRITE (6,20) XP,YP,ZP,XG,YG,X(6),A1,A2,A3
C 20 FORMAT (1X,'SSSS',6F15.7)
      RETURN
      END
C
      SUBROUTINE NONLIN
C
C THIS SUBROUTINE IS USED TO SOLVE NONLINEAR EQUATIONS BY NEWTON-
C RAPHSON METHOD
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
      EPSI,DELTA,NC,NE,NDIM
C
      DO 5 I=1,NC
      CALL FUNC
      WRITE (6,10) I,(X(J),Y(J),J=1,5)
C 10 FORMAT(1X,'***',I5/5(1X,2D15.7__))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
      15 CONTINUE
      GO TO 105
      25 DO 35 J=1,NE
      35 Y1(J)=Y(J)
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (X(J).EQ.0.D0) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
      55 A(K,J)=(Y1(K)-Y(K))/DIFF
      45 CONTINUE
      DO 65 J=1,NE
      65 Y(J)=-Y1(J)
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM,NE,A,Y,IPVT)
      DO 75 J=1,NE
      75 X(J)=X(J)+Y(J)
      5 CONTINUE
      105 RETURN
      END
C
C

```

```

C
C      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C      DECOMPOSES A REAL MATRIX BY GAUSSIAN ELIMINATION,
C      AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C      -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C      M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C      USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C      INITI.. .
C
C      NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING A
C      N    = ORDER OF THE MATRIX
C      A    = MATRIX TO BE TRIANGULARIZED
C
C      OUTPUT.. .
C
C      A    CONTAINS AN UPPER TRIANGULAR MATRIX U AND A PREMUTED
C      VERSION OF A LOWER TRIANGULAR MATRIX L SO THAT
C      (PERMUIATION MATRIX)*A=L*U
C
C      COND = AN ESTIMATE OF THE CONDITION OF A.
C      FOR THE LINEAR SYSTEM A*X = B , CHANGES IN A AND B
C      MAY CAUSE CHANGES COND TIMES AS LARGE IN X.
C      IF COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C      PRECISION. COND IS SET TO 1.0D+32 IF EXACT
C      SINGULARITY IS DETECTED.
C
C      IPVT    = THE PIVOT VECTOR
C      IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C      IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C      WORK SPACE.. TH. VECTOR WORK MUST BE DECLARED AND INCLUDED
C      IN THE CALL. ITS INITI CONTENTS ARE IGNORED.
C      ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C      THE DETERMINANT OF A CAN BE OBTAINED ON OUTPUT BY
C      DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
C      IPVT(N)=1
C      IF (N.EQ.1) GO TO 150
C      NM1=N-1
C
C                  COMPUTE THE 1-NORM OF A .
C      ANORM=0.D0
C      DO 20 J=1,N
C          T=0.D0
C          DO 10 I=1,N
C              T=T+DABS(A(I,J))
C              IF (T.GT.ANORM) ANORM=T
C 10      CONTINUE
C
C                  DO GAUSSIAN ELIMINATION WITH PARTIAL
C                  PIVOTING.
C
C      DO 70 K=1,NM1

```

```

      KP1=K+1
C           FIND THE PIVOT.
      M=K
      DO 30 I=KP1,N
            IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
30    CONTINUE
      IPVT(K)=M
      IF (M.NE.K) IPVT(N)=-IPVT(N)
      T=A(M,K)
      A(M,K)=A(K,K)
      A(K,K)=T
C           SAIP THE ELIMINATION STEP IF PIVOT IS ZERO.
      IF (T.EQ.0.D0) GO TO 70
C
C           COMPUTE THE MULTIPLIERS.
      DO 40 I=KP1,N
40    A(I,K)=-A(I,K)/T
C           INTERCHANGE AND ELIMINATE BY COLUMNS.
      DO 60 J=KP1,N
            T=A(M,J)
            A(M,J)=A(K,J)
            A(K,J)=T
            IF (T.EQ.0.D0) GO TO 60
            DC 50 I=KP1,N
50    A(I,J)=A(I,J)+A(I,K)*T
60    CONTINUE
70    CONTINUE
C
C           COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C           THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C           SMALL SINGULAR VECTOR. THIS INVOLVES SOLVING TWO SYSTEMS
C           OF EQUATIONS, (A-TRANSPOSE)*Y = E AND A*Z = Y WHERE E
C           IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C           ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C           SOLVE (A-TRANSPOSE)*Y = E .
      DO 100 K=1,N
            T=0.D0
            IF (K.EQ.1) GO TO 90
            KM1=K-1
            DO 80 I=1,KM1
80    T=T+A(I,K)*WORK(I)
            90   EK=1.D0
                  IF (T.LT.0.D0) EK=-1.D0
                  IF (A(K,K).EQ.0.D0) GO TO 160
100   WORK(K)=-(EK+T)/A(K,K)
            DO 120 KB=1,NM1
                  K=N-KB
                  T=0.D0
                  KP1=K+1
                  DO 110 I=KP1,N
110   T=T+A(I,K)*WORK(K)
                  WORK(K)=T
                  M=IPVT(K)
                  IF (M.EQ.K) GO TO 120
                  T=WORK(M)
                  WORK(M)=WORK(K)
                  WORK(K)=T

```

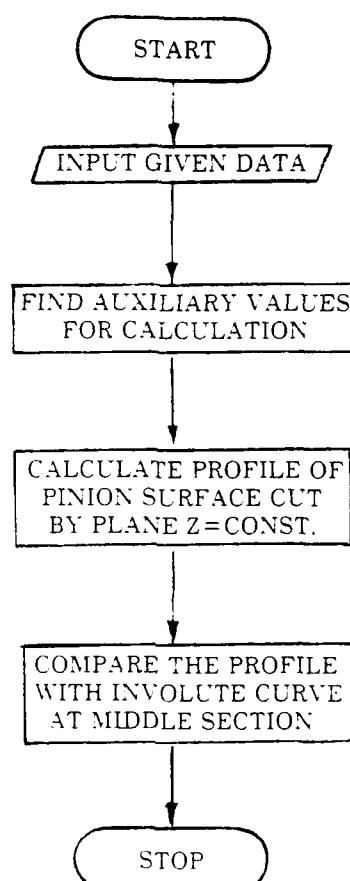
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120 CONTINUE
C
C      YNORM=0.D0
C      DO 130 I=1,N
C      130 YNORM=YNORM+DABS(WORK(I))
C
C              SOLVE A*Z = Y
C              CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
C      ZNORM=0.D0
C      DO 140 I=1,N
C      140 ZNORM=ZNORM+DABS(WORK(I))
C
C      ESTIMATE THE CONDITION.
C      COND=ANORM*ZNORM/YNORM
C      IF (COND.LT.1.D0) COND=1.D0
C      RETURN
C              1-BY-1 CASE..
C      150 COND=1.D0
C          IF (A(1,1).NE.0.D0) RETURN
C
C      EXACT SINGULARITY
C      160 COND=1.0D32
C          RETURN
C          END
C          SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C      SOLVES A LINEAR SYSTEM, A*X = B
C      DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C      -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C      M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C      INPUT..
C
C      NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING A
C      N    = ORDER OF MATRIX
C      A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C      B    = RIGHT HAND SIDE VECTOR
C      IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C      OUTPUT..
C
C      B = SOLUTION VECTOR, X
C
C              DO THE FORWARD ELIMINATION.
C      IF (N.EQ.1) GO TO 50
C      NM1=N-1
C      DO 20 K=1,NM1
C          KP1=K+1
C          M=IPVT(K)
C          T=B(M)
C          B(M)=B(K)
C          B(K)=T
C          DO 10 I=KP1,N

```

```
10    B(I)=B(I)+A(I,K)*T
20 CONTINUE
C                               NOW DO THE BACA SUBSTITUTION.
      DO 40 KB=1,NM1
         KM1=N-KB
         K=KM1+1
         B(K)=B(K)/A(K,K)
         T=-B(K)
         DO 30 I=1,KM1
30    B(I)=B(I)+A(I,K)*T
40 CONTINUE
50 B(1)=B(1)/A(1,1)
      RETURN
      END
```

FLOWCHART FOR PROGRAM II



```

C... ****
C... *
C... *
C... *           PROGRAM II
C... *           SURFACE OF PINION GENERRATED BY CONE CUTTER
C... *
C... *           AUTHORS: FAYDOR LITVIN
C... *           JIAO ZHANG
C... *
C... *
C... *           ****
C... *
C PURPOSE
C
C THIS PROGRAM IS USED TO CALCULATE THE SURFACE OF A PINION WHICH IS
C GENERATED BY CONE CUTTER
C
C NOTE
C
C THIS PROGRAM IS WRITTEN IN FORTRAN 77. IT CAN BE COMPILED BY V
C COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
C IMPLICIT REAL*8(A-H,O-Z)
C DIMENSION NS(15),XP(15,999),YP(15,999),XERROR(999)
C
C DEFINE PARAMETERS USED BY PROGRAMS
C
C (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C     DEVICES
C     IN=5
C     LP=6
C (2) NDEBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
C     NDEBUG=2
C (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C     EQUTIONS;
C     EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C     IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C     RKSTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C     NC, EPSI AND RKSTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C     OR LESS ACCURATE
C     NC=100
C     RKSTA=1.D-3
C     EPSI=1.D-12
C (4) OTHER PARAMETERS(DON'T CHANGE)
C     DR=DATAN(1.D0)/45.D0
C
C DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   1. PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C     RKS=PRESSURE ANGLE(DEGREE); HDC=HEIGHT OF DEDENDUM OF PINION;
C     HAC=HEIRHT OF ADDENDUM OF PINION
C     PN=10.D0
C     N1=20
C     RKS=20.D0*DR
C     HDC=1.D0/PN
C     HAC=1.D0/PN
C   2. TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C     RC=RADIUS OF BOTTOM CIRCLE OF CONE

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```

ALP=80.D0*DR
RC=1.D0
C (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C      NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C      (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
C      N=500
C      NG=2
C      GAMMAI=5.D0
C (4) OUTPUT: ZI=INCREMENT OF TOOTH LENGTH OF CROSS SECTION;
C      NL=NO. OF CROSS SECTIONS WHERE PINION PROFILE IS SIMULATED;
C      N=NO. OF POINTS USED TO DEVIDE TOOL PROFILE IN ANY CROSS
C      SECTION(LARGE N, MORE POINTS IS GOT FOR PINION TOOTH PROFILE)
ZI=0.10D0
NL=3
N=500
C
C DESCRIPTION OF OUTPUT PARAMETERS
C
C      ZI=DISTANCE BETWEEN CROSS SECTION CONSIDERED AND MIDDLE CROSS
C          SECTION
C      NO=OUTPUT NO.
C      Y1=TOOL SURFACE AUXILIARY VARIABLE
C      XP=X COORDINATE OF PINION PROFILE
C      YP=Y COORDINATE OF PINION PROFILE
C      R1=RADIUS OF PINION PROFILE
C      FEE=CORESPONDING PINION SURFACE PARAMETER
C      XSH=AVERAGE DEVIATION SHIFT OF CROSS SECTION PROFILE FROM PROFILE
C          OF MIDDLE SECTION(INVOLUTE CURVE)
C      XPE=MAXIMUM DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE CURVE
C      SDX=STANDARD DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE
C          CURVE
C
C FIND AUXILIARY VALUES FOR CALCULATION
RP=FLOAT(N1)/2.D0/PN
CL=RC/DSIN(ALP)
D=CL**DCOS(ALP)
A1=CL-HDC/DCOS(RKS)
RPU=RP+HAC
DO 5 I=1,NL
Z=FLOAT(I-1)*ZI
YY=D-Z/DTAN(ALP)
WRITE (LP,10) Z
10 FORMAT (1H1/1X,'Z1=',F15.7/1X,'NO',10X,'Y1',13X,'XP',13X,'YP',
      +           13X,'R1',13X,'FEE')
C CALCULATE THE PROFILE OF THE SURFACE CUT BY PLANE Z=CONST
KKK=0
DO 15 J=1,N
Y1=YY+FLOAT(J-1)/FLOAT(N-1)
F=DSORT(1.-(Z/(D-Y1)/DTAN(ALP))**2)
FEE=((D-Y1)**F/DCOS(ALP)-A1*(F**DCOS(ALP)**DCOS(ALP)+DSIN(ALP)**DSIN(
      - ALP))+(RP/(DSIN(ALP)**DCOS(ALP-RKS)-F**DCOS(ALP)**DSIN(ALP-RKS))-
      XP(I,J)=(D-Y1)**(DTAN(ALP)**F**DCOS(ALP-RKS+FEE)-DSIN(ALP-RKS+FEE))
      - A1**DSIN(FEE-RKS)-RP**FEE**DCOS(FEE)+RP**DSIN(FEE)
      YP(I,J)=-(D-Y1)**(DTAN(ALP)**F**DSIN(ALP-RKS+FEE)+DCOS(ALP-RKS+FEE))
      - A1**DCOS(FEE-RKS)+RP**FEE**DSIN(FEE)+RP**DCOS(FEE)
      R1=DSORT(XP(I,J)**2-YP(I,J)**2)
      IF (R1.GT.RPU) THEN
      J=J-1

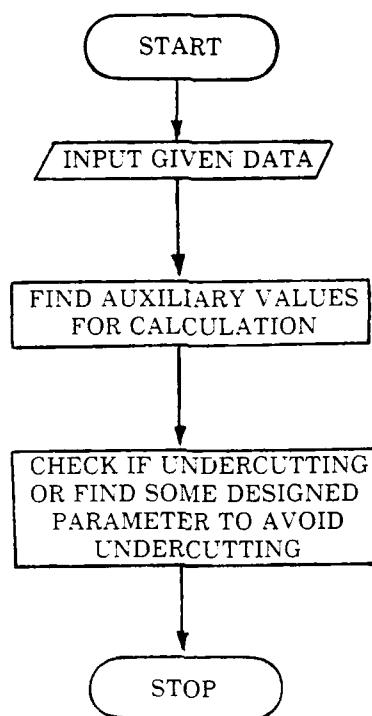
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XP(I,J)=XP(I,JJ)+(XP(I,J)-XP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
YP(I,J)=YP(I,JJ)+(YP(I,J)-YP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
FEE=FEEITEM+(FEE-FEEITEM)/(R1-R1TEMP)*(RPU-R1TEMP)
Y1=Y1ITEM+(Y1-Y1ITEM)/(R1-R1TEMP)*(RPU-R1TEMP)
R1=DSQRT(XP(I,J)**2+YP(I,J)**2)
KKK=1
END IF
WRITE (LP,20) J,Y1,XP(I,J),YP(I,J),R1,FEE
20 FORMAT (1X,I4,4F15.7,F15.7)
IF (KKK.GT.0) GO TO 30
FEETEM=FEE
Y1ITEM=Y1
R1TEMP=R1
15 CONTINUE
30 NS(I)=J-1
IF (I.NE.1) GO TO 55
NS1=NS(1)
GO TO 5
C PREPARATION OF INTERPLORATION
55 NS2=NS(I)
DO 105 L=1,NS2
J=NS2+1-L
IF (YP(1,NS1).GT.YP(I,J)) GO TO 110
105 CONTINUE
110 NS2=J
NREC=2
XERS=0.D0
DO 115 L=1,NS2
DO 125 J=NREC,NS1
IF (YP(1,J).GT.YP(I,L)) GO TO 120
125 CONTINUE
120 NREC=J
J1=J-1
XERROR(L)=XP(I,L)-XP(I,J1)-(YP(I,L)-YP(I,J1))*(XP(I,J)-XP(I,J1))
- / (YP(I,J)-YP(I,J1))
XERS=XERS+XERROR(L)
115 CONTINUE
XERS=-XERS/FLOAT(NS2)
XPE=0.D0
SDX=0.D0
IF (NDEBUG.GT.2) WRITE (LP,80)
80 FORMAT (1H1,13X,'NO.',4X,'DIVIATION VALUE')
DO 45 L=1,NS2
XERROR(L)=XERROR(L)+XERS
IF (NDEBUG.GT.2) WRITE (LP,50) L,XERROR(L)
50 FORMAT (13X,13,2F15.7)
IF (DABS(XERROR(L)).GT.XPE) XPE=DABS(XERROR(L))
SDX=SDX+XERROR(L)**2
45 CONTINUE
SDX=DSORT(SDX/FLOAT(NS2))
WRITE (LP,40) XERS,XPE,SDX
40 FORMAT (//1X,'XSH=' ,E15.7,5X,'XPE=' ,E15.7,5X,'SDX=' ,E15.7)
IF (NDEBUG.GT.2) WRITE (LP,60) NS1,NS2
60 FORMAT (//5X,'NS1=' ,16.6X,'NS2=' ,16)
5 CONTINUE
STOP
END

```

FLOWCHART FOR PROGRAM III



```

C... ****
C... *
C... *
C... * PROGRAM III
C... * UDDERCUTTING CONDITION FOR PINION GENERRATED
C... * BY CONE CUTTER
C... *
C... * AUTHOR: FAYDOR LITVIN
C... * JIAO ZHANG
C... *
C... *
C... ****
C
C PURPOSE
C
C THIS PROGRAM IS USED TO FIND THE UDDERCUTTING CONDITIONS FOR A
C PINION GENERATED BY CONE CUTTER
C
C NOTE
C
C THIS PROGRAM IS WRITTEN IN FORTRAN 77. IT CAN BE COMPILED BY V
C COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
C IMPLICIT REAL*8(A-H,O-Z)
C
C DEFINE PARAMETERS USED BY PROGRAMS
C
C (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C DEVICES
    IN=5
    LP=6
C (2) NDEBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
    NDEBUG=2
C (3) OTHER PARAMETERS(DON'T CHANGE)
    DR=DATAN(1.D0)/45.D0
C
C DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C Rksi=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C TL=TOOTH LENGTH
    PN=10.D0
    N1=20
    Rksi=20.D0*DR
    HDC=1.D0/PN
    TL=6.D0/PN
C (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C RC=RADIUS OF BOTTOM CIRCLE OF CONE
    ALPHA=80.D0*DR
    RC=1.D0
C (3) PROBLEM: NPROB=ID NO. OF PROBLEM (-1=GIVEN N1 AND HDC, FIND IF
C UDDERCUTTING OCCUR; 0=GIVEN N1, FIND MAXIMUM HDC WITHOUT UDDER-
C CUTTING; 1=GIVEN HDC, FIND MINIMUM N1 WITHOUT UDDERCUTTING);
C N=NO. OF THETA VALUES USED CALCULATION (BETWEEN THETAS CORRE-
C PONDING TO MIDDLE SECTION AND EDGE SECTION OF PINION)
    NPROB= 1
    N=11
C

```

```

C DESCRIPTION OF OUTPUT
C
C   OUTPUT IS A STATEMENT BASED ON THE PROBLEM WITHOUT ANY LITERAL
C   PARAMETER
C
C FIND AUXILIARY VALUES FOR CALCULATION
CL=RC/DSIN(ALPHA)
SA=DSIN(ALPHA)
CA=DCOS(ALPHA)
SK=DSIN(RKSI)
CK=DCOS(RKSI)
SAK=DSIN(ALPHA-RKSI)
CAK=DCOS(ALPHA-RKSI)
THEMAX=DARSIN(TL/2.0/SA)
IF (NPROB.NE.0) THEN
A1=(CL-HDC/CK)*CK
ACL=A1/CL
END IF
IF (NPROB.LT.1) THEN
RP=FLOAT(N1)/2./PN
RCL=RP/CL
END IF
WRITE (LP,90)
90 FORMAT (1H1)
IF (NPROB) 5,15,25
C CHECK IF UNDERCUTTING OCCURS
5 IF (NDBUG.LT.1) WRITE (LP,10)
10 FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
+           'B**2-4.*A*C',3X,'U1/(RC/SIN(ALPHA))')
UMIN=5.D0
DO 45 I=1,N
THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
ST=DSIN(THE)
CT=DCOS(THE)
COE1=CA*CAK+CT*SA*SAK
COE2=SA*CAK-CT*CT*CA*SAK
COE3=SA*CAK-CT*CA*SAK
COE4=ST*ST*SA*CA
A=COE1*COE2
B=-ACL*(COE2+COE1*COE4)-RCL*COE3**3
C=ACL*ACL*COE4
DD=B*B-4.*A*C
IF (DD.GE.0.) THEN
U1CL=(-B+DSQRT(DD))/2./A
IF (UMIN.GT.U1CL) UMIN=U1CL
U2CL=(-B-DSQRT(DD))/2./A
IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD,U1CL
100 FORMAT (1X,I4,8F15.7)
ELSE
IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD
END IF
45 CONINUE
IF (UMIN.LT.1.D0) WRITE (LP,110)
110 FORMAT (///1X,'UNDERCUTTING WILL OCCUR FOR YOUR DESIGN')
IF (UMIN.GE.1.D0) WRITE (LP,120)
120 FORMAT (///1X,'UNDERCUTTING WILL NOT OCCUR FOR YOUR DESIGN')
GO TO 35
C DETERMINE THE MAXIMUM ADDENDUM HEIGHT OF RACA CUTTER

```

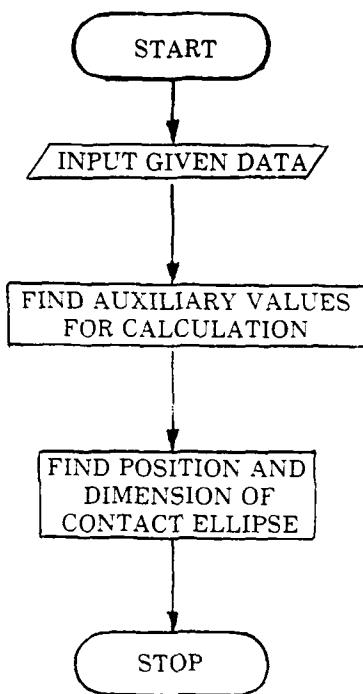
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15 IF (NDBUG.LT.1) WRITE (LP,20)
20 FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
+          'B**2-4.*A*C',3X,'ALLOWED RATIO OF HDC/(1/PN)')
      UMIN=5.D0
      DO 55 I=1,N
      THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
      ST=DSIN(THE)
      CT=DCOS(THE)
      COE1=CA*CAK+CT*SA*SAK
      COE2=SA*CAK-CT*CT*CA*SAK
      COE3=SA*CAK-CT*CA*SAK
      COE4=ST*ST*SA*CA
      C=COE1*COE2-RCL*COE3**3
      B=-(COE2+COE1*COE4)
      A=COE4
      DD=B*B-4.*A*C
      IF (DD.GE.0.) THEN
        IF (A.NE.0.) THEN
          U1CL=(-B-DSQRT(DD))/2./A
          U2CL=(-B+DSQRT(DD))/2./A
        ELSE
          U1CL=-C/B
          U2CL=0.
        END IF
        U1CL=(CK-U1CL)*CL*PN
        U2CL=(CK-U2CL)*CL*PN
        IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD,U1CL
      ELSE
        IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD
      END IF
      IF (UMIN.GT.U1CL) UMIN=U1CL
55 CONTINUE
      WRITE (LP,200) UMIN
200 FORMAT (///1X,'TO AVOID UDDERCUTTING, IT IS NECESSARY TO KEEP DEDE
+NDUM OF PINION <=',F10.7,'/PN')
      GO TO 35
C DETERMINE THE MINIMUM NO. OF TEETH FOR UNUNDERCUTTING
25 IF (NDBUG.LT.1) WRITE (LP,30)
30 FORMAT (1H1/3X,'NO',7X,'THETA',7X,'NO. OF TEETH')
      RNMAX=0.
      DO 65 I=1,N
      THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
      ST=DSIN(THE)
      CT=DCOS(THE)
      COE1=CA*CAK+CT*SA*SAK
      COE2=SA*CAK-CT*CT*CA*SAK
      COE3=SA*CAK-CT*CA*SAK
      COE4=ST*ST*SA*CA
      RR=(COE1*COE2-ACL*(COE2+COE1*COE4)-ACL*ACL*COE4)/COE3**3
      RN=2.*RR*PN*CL
      IF (RNMAX.LT.RN) RNMAX=RN
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,RN
65 CONTINUE
      WRITE (LP,300) RNMAX
300 FORMAT (/// 1X,'WITHOUT UDDERCUTTING, MINIMUM TOOTH NO. OF PINION
+ IS:',F11.7)
      35 WRITE (LP,400)
400 FORMAT(1H1)

```

STOP  
END

FLOWCHART FOR PROGRAM IV



```

C... **** * ***** * ***** * ***** * ***** * ****
C... *
C... *
C... *
C... * PROGRAM IV *
C... * CONTACT ELLIPSIS FOR PINION GENERATED BY CONE *
C... * CUTTER IN MESHING WITH REGULAR GEAR *
C... *
C... * AUTHORS: FAYDOR LITVIN *
C... * JIAO ZHANG *
C... *
C... *
C... **** * ***** * ***** * ***** * ***** * ****
C
C PURPOSE
C
C THIS PROGRAM IS USED TO CALCULATE THE CONTACT ELLIPSIS WHEN PINION
C GENERATED BY CONE CUTTER IN MESHING WITH REGULAR INVOLUTE GEAR
C
C NOTE
C
C THIS PROGRAM IS WRITTEN IN FORTRAN 77. IT CAN BE COMPILED BY V
C COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
C IMPLICIT REAL*8(A-H,O-Z)
C
C DEFINE PARAMETERS USED BY PROGRAMS
C
C (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C     DEVICES
    IN=5
    LP=6
C (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
    NDBUG=2
C (3) OTHER PARAMETERS(DON'T CHANGE)
    DR=DATAN(1.D0)/45.D0
C
C DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C     RMPC=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C     RKSI=PRESSURE ANGLE
    PN=10.D0
    N1=20
    RMPC=2.D0
    RKSI=20.D0*DR
C (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C     RC=RADIUS OF BOTTOM CIRCLE OF CONE;
    ALPHA=89.5D0*DR
    RC=1.D0
C (3) DEFORMATION: DEL=CONTACT DEFORMATION AT CONTACT POINT
    DEL=4.D-4
C (4) OUTPUT: PD=RECIPROCAL OF INCREMENT OF S VALUE(S IS THE DISTANCE
C     CONTACT POINT AND INSTANTANEOUS CENTER)
    PD=100.D0
C
C DESCRIPTION OF OUTPUT PARAMETER
C
C S=DISTANCE BETWEEN CONTACT POINT AND INSTANTANEOUS CENTER

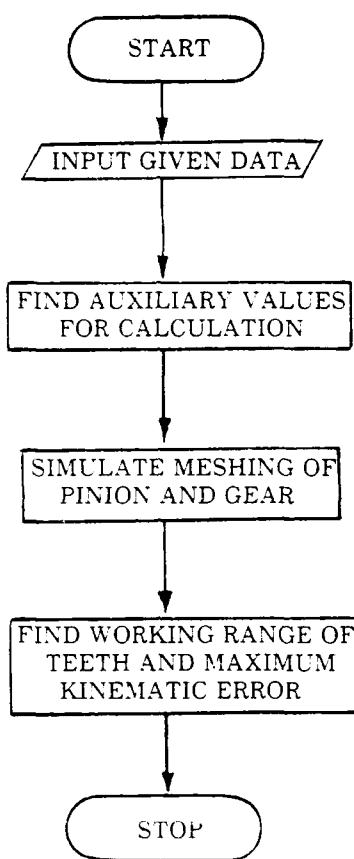
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C   R1=PINION RADIUS OF CONTACT POINT
C   FEE=PINION ROTATION ANGLE CORRESPONDING TO CONTACT POINT
C   A=LENGTH OF HALF SHORT AXIS OF CONTACT ELLIPSE
C   B=LENGTH OF HALF LONG AXIS OF CONTACT ELLIPSE(ALONG DIRECTION OF
C     GEAR TOOTH LENGTH)
C
C   FIND AUXILIARY VALUES FOR CALCULATION
  RP=FLOAT(N1)/2./PN
  SA=DSIN(ALPHA)
  CA=DCOS(ALPHA)
  SK=DSIN(RKSI)
  CK=DCOS(RKSI)
  RL=RC/SA
  SMAX=(DSQRT(((1.D0+2.D0/FLOAT(N1))/CK)**2-1.D0)-SK/CK)*CK*RP
  NA=IDINT(SMAX*PD+0.5D0)
  SMAX=FLOAT(NA)/PD
  SMIN=(DSQRT(((1.+2./FLOAT(N1)/RMPG)/CK)**2-1.))-SK/CK)*CK*RP*RMPG
  NB=-IDINT(SMIN*PD+0.5D0)
  SMIN=FLOAT(NB)/PD
  N=1+NA-NB
  WRITE (LP,10) RP,N1,RMPG
10 FORMAT (1H1, //5X, 'RADIUS OF THE PITCH CIRCLE OF PINION (RP):',
  -F15.7//5X, 'TOOTH NO. OF PINION(N1):', I8//5X, 'RATIO OF ANGULAR VELO
  -CITY (OMEGA OF PINION/OMEGA OF GEAR):', F15.7///1X, 'OUTPUT DESIGNAT
  -ION: S=DISTANCE BETWEEN CONTACT POINT AND INSTANTANEOUS CENTER'/
  +21X, 'R1=PINION RADIUS OF CONTACT POINT; FEE=PINION ROTATION CORRE
  -PONDING TO CONTACT POINT'/21X, 'A=LENGTH OF HALF SHORT AXIS; B=LENG
  -TH OF HALF LONG AXIS(ALONG DIRECTION OF GEAR TOOTH LENGTH)'/////
  +10X, 'S', 14X, 'R1', 10X, 'FEE1(D)', 11X, 'A', 14X, 'B', 13X, 'B/A'//)
C   FIND POSITION AND DIMENSION OF CONTACT ELLIPSE
  DO 5 L=1,N
  S=SMIN+(SMAX-SMIN)/FLOAT(N-1)*FLOAT(L-1)
  A=DSQRT(DEL*RP*(SK+S/RP)*(RMPG*SK-S/RP)**2.D0/(1.D0+RMPG)/SK)
  B=DSQRT(DEL*2.D0*SA/CA*(RL-2.D0*RP/CK/FLOAT(N1)-S*SK/CK))
  C=B/A
  FEE=S/RP/CK
  R1=RP*DSQRT(1.D0+(FEE*CK)**2+2.D0*FEE*CK*SK)
  FEE=FEE/DR
  WRITE (LP,20) S,R1,FEE,A,B,C
20 FORMAT(1X,6F15.7)
  5 CONTINUE
  WRITE (LP,30)
30 FORMAT (1H1)
  STOP
  END

```

FLOWCHART FOR PROGRAM V

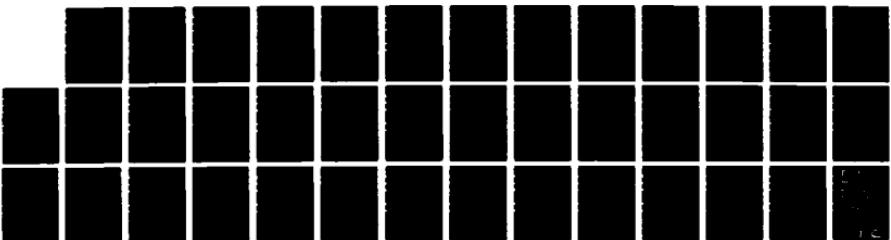


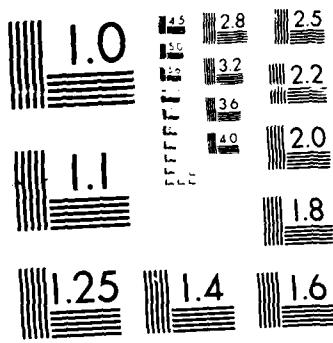
```

C... ****
C... *
C... *
C... *
C... * PROGRAM V
C... * KINEMATIC ERROR OF THE PINION GENERATED BY CONE
C... * CUTTER MESHING WITH A MISALIGNED REGULUAR GEAR
C... *
C... * AUTHORS: FAYDOR LITVIN
C... * JIAO ZHANG
C... *
C... *
C... ****
C
C PURPOSE
C
C THIS PROGRAM IS USED TO CALCULATE THE KINEMATIC ERROR OF A PINION
C GENERATEED BY CONE CUTTER MESHING WITH A MISALIGNED REGULAR
C INVOLUTE GEAR
C
C NOTE
C
C THIS PROGRAM IS WRITTEN IN FORTRAN 77. IT CAN BE COMPILED BY V
C COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
C IMPLICIT REAL*8(A-H,O-Z)
C DIMENSION Z(99),ERR(99),ERROR(99)
C COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
C - EPSI,DELTA,NC,NE,NDIM
C COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,
C - C,S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R
C
C DEFINE PARAMETERS USED BY PROGRAMS
C
C (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C DEVICES
C     IN=5
C     LP=6
C (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
C     NDBUG=2
C (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C EQUATIONS;
C     EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C     IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF  $F(X)=0$ );
C     DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C     NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C     OR LESS ACCURATE
C     NC=100
C     DELTA=1.D-3
C     EPSI=1.D-12
C (4) OTHER PARAMETERS(DON'T CHANGE)
C     NDIM=10
C     NE=5
C     DR=DATAN(1.D0)/45.D0
C
C DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C     RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C     RKSI=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C     COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)

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AD-A157 797 - SPUR GEARS: OPTIMAL GEOMETRY METHODS FOR GENERATION AND 272  
TOOTH CONTACT ANALYSIS (U) ILLINOIS UNIV AT CHICAGO CIRCLE  
DEPT OF MECHANICAL ENGINEERING F L LITVIN ET AL  
UNCLASSIFIED APR 88 NASA-CR-4135 NAG3-655 F/G 13/9 NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS 1962-A

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PN=10.D0
N1=20
RMPC=2.D0
RKS1=20.D0*DR
HDC=1.D0/PN
COE=1.005D0
C (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C     RC=RADIUS OF BOTTOM CIRCLE OF CONE
C     R=RADIUS OF ARC(FOR EXACT CONE CHOOSE R >1.D6*N1/2/PN)
C     ALPHA=80.D0*DR
RC=1.D0
R=1.0D3
C (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C     NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C     (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
NMIS=1
NG=2
GAMMAI=5.D0
C (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREES)
FEEI=1.0D0*DR
C
C DESCRIPTION OF OUTPUT PARAMETERS
C
C     FEE1=ROTATION ANGLE OF PINION
C     FEE2=ROTATION ANGLE OF GEAR
C     RP=RADIUS OF PINION CONTACT POINT
C     RG=RADIUS OF GEAR CONTACT POINT
C
C FIND AUXILIARY VALUES FOR CALCULATION
RP=FLOAT(N1)/2./PN
RG=RP*RMPC
C=(RP+RG)*COE
CA=DCOS(ALPHA)
SA=DSIN(ALPHA)
CK=DCOS(RKS1)
SK=DSIN(RKS1)
SAK=DSIN(ALPHA-RKS1)
CAK=DCOS(ALPHA-RKS1)
RL=RC/SA
AI=RL-HDC/CK
NCOEF=360.D0*DR/FEEI/FLOAT(N1)+0.3D0
N=NCOEF*2+1
NT=N-NCOEF
FII=360.D0/RMPC/FLOAT(N1)
C DEFINE MATRIX DESCRIBING MISALIGNMENT AND OUTPUT ASSEMBLING CONDITION
DO 5 I=1,3
DO 5 J=1,3
S(I,J)=0.D0
IF (I.EQ.J) S(I,J)=1.D0
5 CONTINUE
DO 15 LL=1,NG
GAMMA=GAMMAI*FLOAT(LL-0)/60.D0*DR
CG=DCOS(GAMMA)
SG=DSIN(GAMMA)
GAMMA=GAMMA/DR*60.D0
IF (NMIS.EQ.1) THEN
WRITE (LP,10) COE,GAMMA
10 FORMAT (1H1,///,1X,'C=' ,F5.3,'*(RP+RG);      CROSSING ANGLE=' ,

```

```

+      F5.1,'(M)')
S(1,1)=CG
S(1,3)=-SG
S(3,1)=SG
S(3,3)=CG
ELSE
  WRITE (LP,20) COE,GAMMA
20 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG);    INTERSECTING ANGLE=',
+          F5.1,'(M)')
  S(2,2)=CG
  S(2,3)=-SG
  S(3,2)=SG
  S(3,3)=CG
END IF
C SIMULATING MESHING OF PINION AND GEAR(L=1 FOR FINDING INITIAL
C ROTATION CORRESPONDING TO 0 PINION ROTATION) AND OUTPUT RESULTS
DO 25 L=1,2
DO 35 I=1,4
35 X(I)=0.D0
DO 45 I=1,N
  X(7)=FEEI*FLOAT(I-(N+1)/2)
  IF (L.EQ.1) X(7)=0.D0
  X(5)=DARSIN(RP*X(7)*SK/R)
  X(2)=X(7)/RMPG
  SF=DSIN(X(7))
  CF=DCOS(X(7))
  CALL NONLIN
  X(8)=X(2)+X(3)
  IF (L.EQ.1) THEN
    XIN=X(8)
    WRITE (LP,30)
30 FORMAT (///8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
+          'RP',13X,'RG')
    GO TO 25
  END IF
  X(8)=X(8)-XIN
  X(7)=X(7)/DR
  X(8)=X(8)/DR
  X(9)=(X(8)-X(7)/RMPG)*3600.D0
  Z(I)=X(8)
  ERR(I)=X(9)
  WRITE (LP,40) (X(J),J=7,11)
40 FORMAT (1X,5F15.7,F15.7)
45 CONTINUE
  WRITE (LP,50)
50 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:/')
  DO 55 I=1,NT
    X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
    X(8)=X(7)+FII
    KK=I+NCOEF
    ANGLE=Z(KK)-Z(I)
    ERROR(I)=(ANGLE-FII)*3600.D0
    WRITE (LP,60) X(7),X(8),ANGLE
60 FORMAT (1X,(' ',F7.2,'----',F7.2,':',F15.7,F15.7)
55 CONTINUE
  DO 65 I=1,NT
    ATEMP2=ERROR(I)
    IF (I.NE.1) THEN

```

```

        IF (ATEMP1.GT.0.D0.AND.ATEMP2.LE.0.D0) GOTO 75
END IF
ATEMP1=ATEMP2
65 CONTINUE
WRITE (LP,70)
70 FORMAT (//1X,'MESHING IS DISCONTINOUS')
GOTO 25
75 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
EMAX=0.D0
EMIN=0.D0
DO 85 J=1,NCOEF
KS=I+J-1
ET=ERR(KS)
IF (ET.LT.EMIN) EMIN=ET
IF (ET.GT.EMAX) EMAX=ET
85 CONTINUE
ET=EMAX-EMIN
KK=I+NCOEF
WRITE (LP,80) Z(I),Z(KK),ET
80 FORMAT (//1X,'WORKING RANGE FOR THE GEAR TOOTH: ',F7.2,'----',
         F7.2/1X,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,' (S)')
25 CONTINUE
15 CONTINUE
WRITE (LP,90)
90 FORMAT(1H1)
STOP
END

C
      SUBROUTINE FUNC
C
C THIS SUBROUTINE IS USED TO GENERATE FIVE FUNCTIONS, THAT IS, AT
C CONTACT POINT, POSITION VECTOR AND NORMAL OF PINION AND GEAR
C MUST COINCIDE
IMPLICIT REAL*8(A-H,O-Z)
COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
               EPSI,DELTA,NC,NE,NDIM
COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,
               C,S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R
CT=DCOS(X(4))
ST=DSIN(X(4))
CFFE=DCOS(X(3))
SFEE=DSIN(X(3))
CFEK=DCOS(X(3)+RKSI)
SFEK=DSIN(X(3)+RKSI)
CAB=DCOS(ALPHA+X(5))
SAB=DSIN(ALPHA+X(5))
CAB2=DCOS(ALPHA+X(5)/2.D0)
SAB2=DSIN(ALPHA+X(5)/2.D0)
SB=DSIN(X(5))
SB2=DSIN(X(5)/2.D0)
A2=A1*SA*(CT-1)
X(6)=(A2*SAB-R*SB*CI)/(SAB*CAK-CI*CAB*SAK)/RP
CFP=DCOS(X(6))
SFP=DSIN(X(6))
CAKF=DCOS(ALPHA-RKSI+X(6))
SAKF=DSIN(ALPHA-RKSI+X(6))
A3=2.D0*R*SB2
T3=CAB2*SAKF-CT*SAB2*CAKF

```

```

XP=A3*(CAB2*SAKF-CT*SAB2*CAKF)+A2*CAKF-RP*X(6)*CFP+RP*SFP
YP=A3*(CAB2*CAKF+CT*SAB2*SAKF)-A2*SAKF+RP*X(6)*SFP+RP*CFP
ZP=(A1*SA-A3*SAB2)*ST
RNXP=CT*CAB*CAKF+SAB*SAKF
RNYP=-CT*CAB*SAKF+SAB*CAKF
RNZP=ST*CAB
XG=RG*SFee+RG*X(2)*CK*CFEK
YG=-RG*CFEE+RG*X(2)*CK*SFEK
ZG=X(1)
RNXG=CFEK
RNYG=SFEK
RNZG=0.D0
C T1=CF*XP+SF*YP
C T2=S(1,1)*XG+S(1,2)*YG+S(1,3)*ZG
C WRITE (6,50) T1,T2,XP,YP,X(6)
C 50 FORMAT (1X,'T1=',F15.7,5X,'T2=',F15.7,2F15.7)
Y(1)=CF*XP+SF*YP-S(1,1)*XG-S(1,2)*YG-S(1,3)*ZG
Y(2)=CF*YP-SF*XP-S(2,1)*XG-S(2,2)*YG-S(2,3)*ZG-C
Y(3)=ZP-S(3,1)*XG-S(3,2)*YG-S(3,3)*ZG
Y(4)=CF*RNYP-SF*RNXP-S(2,1)*RNXG-S(2,2)*RNYG-S(2,3)*RNZG
Y(5)=RNZP-S(3,1)*RNXG-S(3,2)*RNYG-S(3,3)*RNZG
X(10)=DSQRT(XP*XP+YP*YP)
X(11)=DSQRT(XG*XG+YG*YG)
C WRITE (6,20) XP,YP,ZP,XG,YG,X(6),A1,A2,A3
C 20 FORMAT (1X,'SSSS',6F15.7)
RETURN
END
C
SUBROUTINE NONLIN
C
C THIS SUBROUTINE IS USED TO SOLVE NONLINEAR EQUATIONS BY NEWTON-
C RAPHSON METHOD
IMPLICIT REAL*8(A-H,O-Z)
COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
+ EPSI,DELTA,NC,NE,NDIM
C
DO 5 I=1,NC
CALL FUNC
C WRITE (6,10) I,(X(J),Y(J),J=1,5)
C 10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
DO 15 J=1,NE
IF (DABS(Y(J)).GT.EPSI) GO TO 25
15 CONTINUE
GO TO 105
25 DO 35 J=1,NE
35 Y1(J)=Y(J)
DO 45 J=1,NE
DIFF=DABS(X(J))*DELTA
IF (X(J).EQ.0.D0) DIFF=DELTA
XMAM=X(J)
X(J)=X(J)-DIFF
CALL FUNC
X(J)=XMAM
DO 55 K=1,NE
55 A(K,J)=(Y1(K)-Y(K))/DIFF
45 CONTINUE
DO 65 J=1,NE
65 Y(J)=-Y1(J)

```

```

CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
CALL SOLVE (NDIM,NE,A,Y,IPVT)
DO 75 J=1,NE
75 X(J)=X(J)+Y(J)
5 CONTINUE
105 RETURN
END

C
C
C      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
C      IMPLICIT REAL*8(A-H,O-Z)
C      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C      DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C      AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C      -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C      M. A. MALCOLM, AND C. B. MOLES (PRENTICE-HALL, 1977)
C
C      USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C      IN1UT..
C
C      NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C      N    = ORDER OF THE MATRIX
C      A    = MATRIX TO BE TRIANGULARIZED
C
C      OUTPUT..
C
C      A    CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C      VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C      (PERMUTATION MATRIX)*A=L*U
C
C      COND = AN ESTIMATE OF THE CONDITION OF  A.
C      FOR THE LINEAR SYSTEM A*X = B , CHANGES IN  A  AND  B
C      MAY CAUSE CHANGES COND TIMES AS LARGE IN  X.
C      IF COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C      PRECISION.  COND IS SET TO 1.0D+32 IF EXACT
C      SINGULARITY IS DETECTED.
C
C      IPVT     = THE PIVOT VECTOR
C      IPVT(K)  = THE INDEX OF THE K-TH PIVOT ROW
C      IPVT(N)  = (-1)**(NUMBER OF INTERCHANGES)
C
C      WORK SPACE..  THE VECTOR WORK MUST BE DECLARED AND INCLUDED
C      IN THE CALL.  ITS IN1UT CONTENTS ARE IGNORED.
C      ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C      THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C      DET(A) = IPVT(N) * A(1,1) ^ A(2,2) * ... * A(N,N) .
C
C      IPVT(N)=1
C      IF (N.EQ.1) GO TO 150
C      NM1=N-1
C                  COMPUTE THE 1-NORM OF  A .
C      ANORM=0.00

```

```

      DO 20 J=1,N
      T=0.D0
      DO 10 I=1,N
10      T=T+DABS(A(I,J))
      IF (T.GT.ANORM) ANORM=T
20      CONTINUE
C               DO GAUSSIAN ELIMINATION WITH PARTIAL
C               PIVOTING.
C               DO 70 K=1,NM1
C               KP1=K+1
C               FIND THE PIVOT.
C               M=K
C               DO 30 I=KP1,N
C               IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
30      CONTINUE
C               IPVT(K)=M
C               IF (M.NE.K) IPVT(N)=-IPVT(N)
C               T=A(M,K)
C               A(M,K)=A(K,K)
C               A(K,K)=T
C               SAIP THE ELIMINATION STEP IF PIVOT IS ZERO.
C               IF (T.EQ.0.D0) GO TO 70
C               COMPUTE THE MULTIPLIERS.
C               DO 40 I=KP1,N
40      A(I,K)=-A(I,K)/T
C               INTERCHANGE AND ELIMINATE BY COLUMNS.
C               DO 60 J=KP1,N
C               T=A(M,J)
C               A(M,J)=A(K,J)
C               A(K,J)=T
C               IF (T.EQ.0.D0) GO TO 60
C               DO 50 I=KP1,N
50      A(I,J)=A(I,J)+A(I,K)*T
60      CONTINUE
70      CONTINUE
C               COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C               THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C               SMALL SINGULAR VECTOR. THIS INVOLVES SOLVING TWO SYSTEMS
C               OF EQUATIONS, (A-TRANSPOSE)*Y = E AND A*Z = Y WHERE E
C               IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C               ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C               SOLVE (A-TRANSPOSE)*Y = E .
DO 100 K=1,N
      T=0.D0
      IF (K.EQ.1) GO TO 90
      KM1=K-1
      DO 80 I=1,KM1
80      T=T+A(I,K)*WORK(I)
90      EK=1.D0
      IF (T.LT.0.D0) EK=-1.D0
      IF (A(K,K).EQ.0.D0) GO TO 160
100     WORK(K)=-EK+T/A(K,K)
      DO 120 KB=1,NM1
      K=N-KB
      T=0.D0

```

```

      KP1=K+1
      DO 110 I=KP1,N
110   T=T+A(I,K)*WORK(K)
      WORK(K)=T
      M=IPVT(K)
      IF (M.EQ.K) GO TO 120
      T=WORK(M)
      WORK(M)=WORK(K)
      WORK(K)=T
120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
130   YNORM=YNORM+DABS(WORK(I))
C
      SOLVE A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
     ZNORM=0.D0
      DO 140 I=1,N
140   ZNORM=ZNORM+DABS(WORK(I))
C
      ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C
      1-BY-1 CASE..
150  COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
C
      EXACT SINGULARITY
160  COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C SOLVES A LINEAR SYSTEM, A*X = B
C DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C     M. A. MALCOLM, AND C. B. MOLES (PRENTICE-HALL, 1977)
C
C INIT..
C
C     NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING A
C     N    = ORDER OF MATRIX
C     A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C     B    = RIGHT HAND SIDE VECTOR
C     IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C OUTPUT..
C
C     B = SOLUTION VECTOR, X
C
C     DO THE FORWARD ELIMINATION.

```

```

IF (N.EQ.1) GO TO 50
NM1=N-1
DO 20 K=1,NM1
  KP1=K+1
  M=IPVT(K)
  T=B(M)
  B(M)=B(K)
  B(K)=T
  DO 10 I=KP1,N
10  B(I)=B(I)+A(I,K)*T
20 CONTINUE
C                               NOW DO THE BACK SUBSTITUTION.
DO 40 KB=1,NM1
  KM1=N-KB
  K=KM1+1
  B(K)=B(K)/A(K,K)
  T=-B(K)
  DO 30 I=1,KM1
30  B(I)=B(I)+A(I,K)*T
40 CONTINUE
50 B(1)=B(1)/A(1,1)
RETURN
END

```

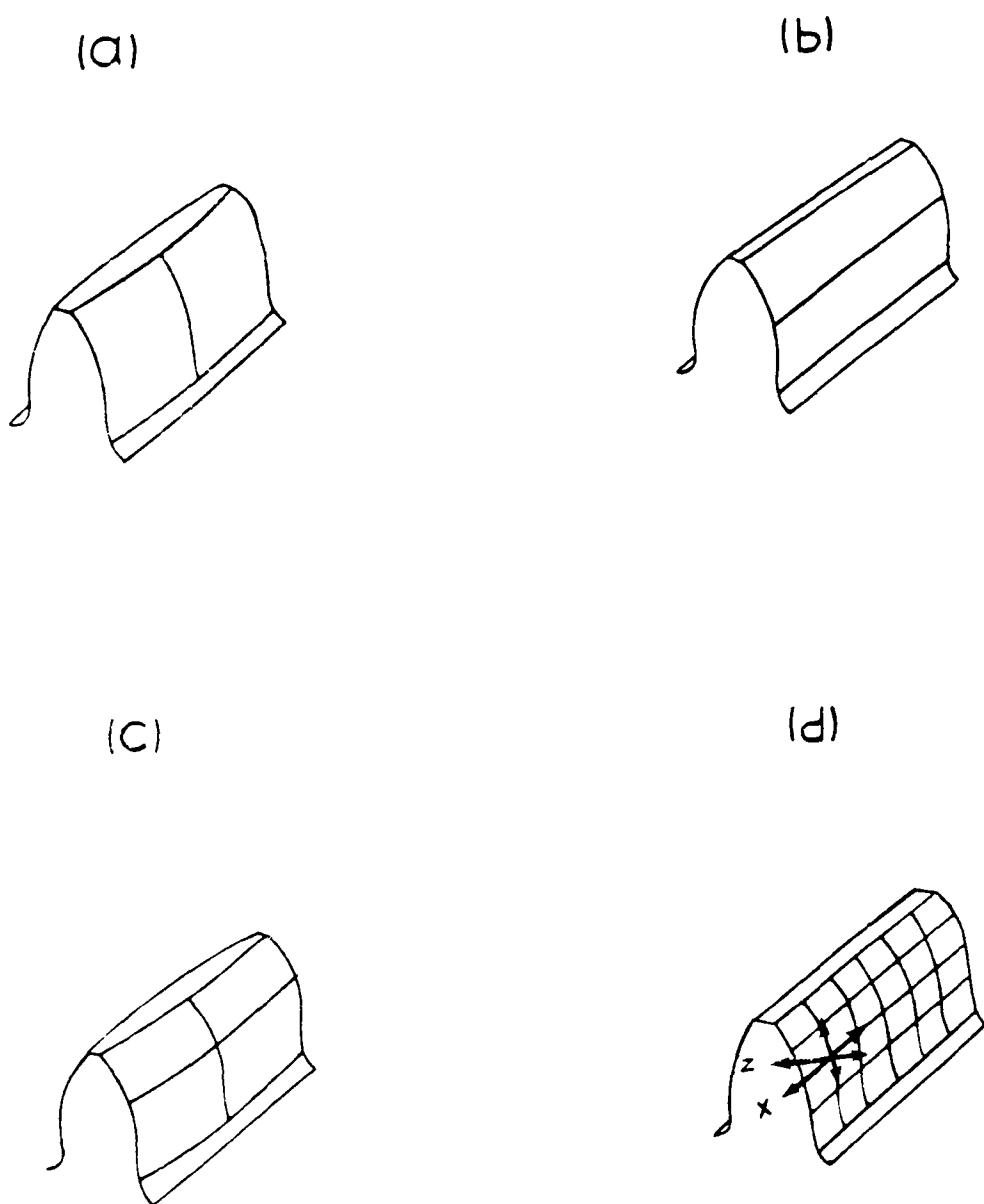
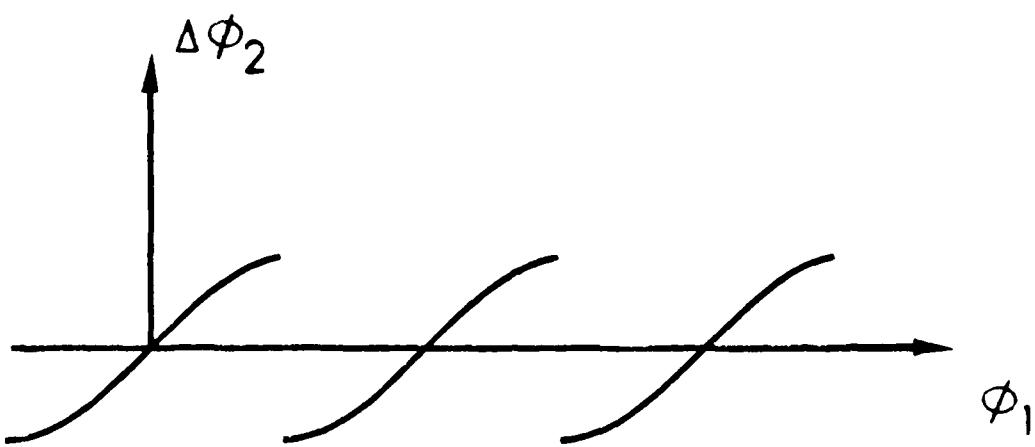


FIG. 1.1

(a)



(b)

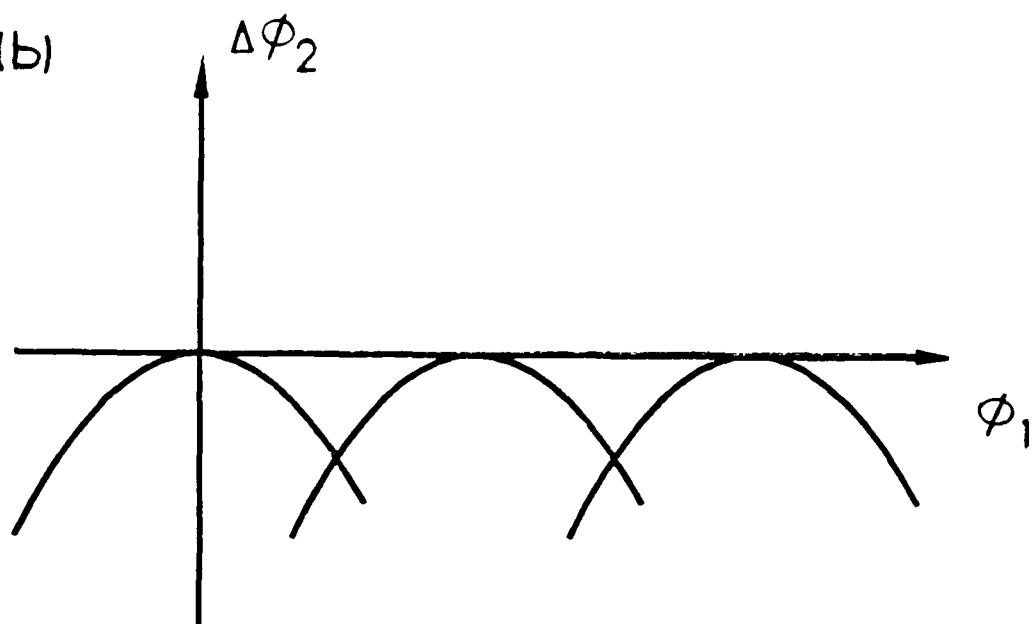


FIG.1.2

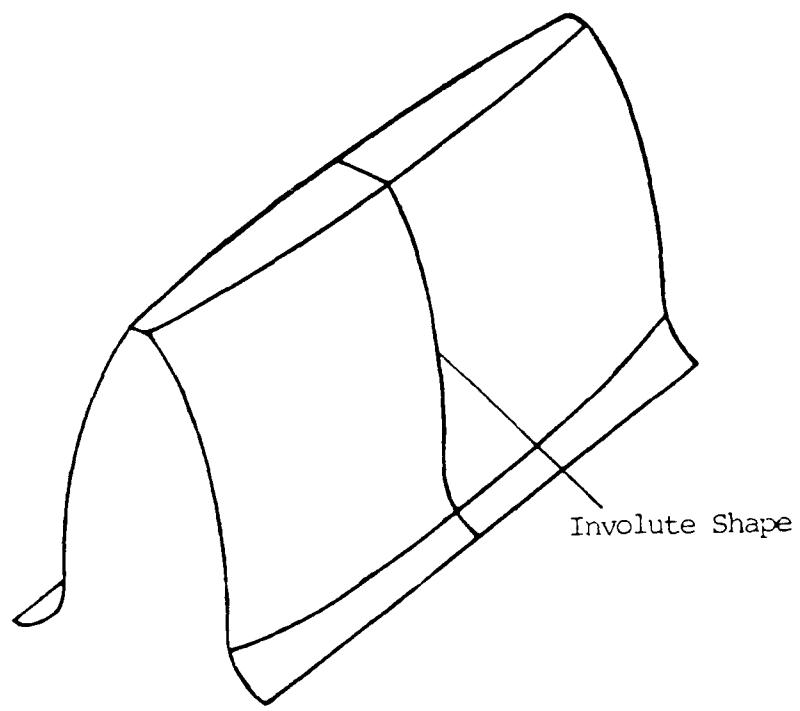


FIG.1.3

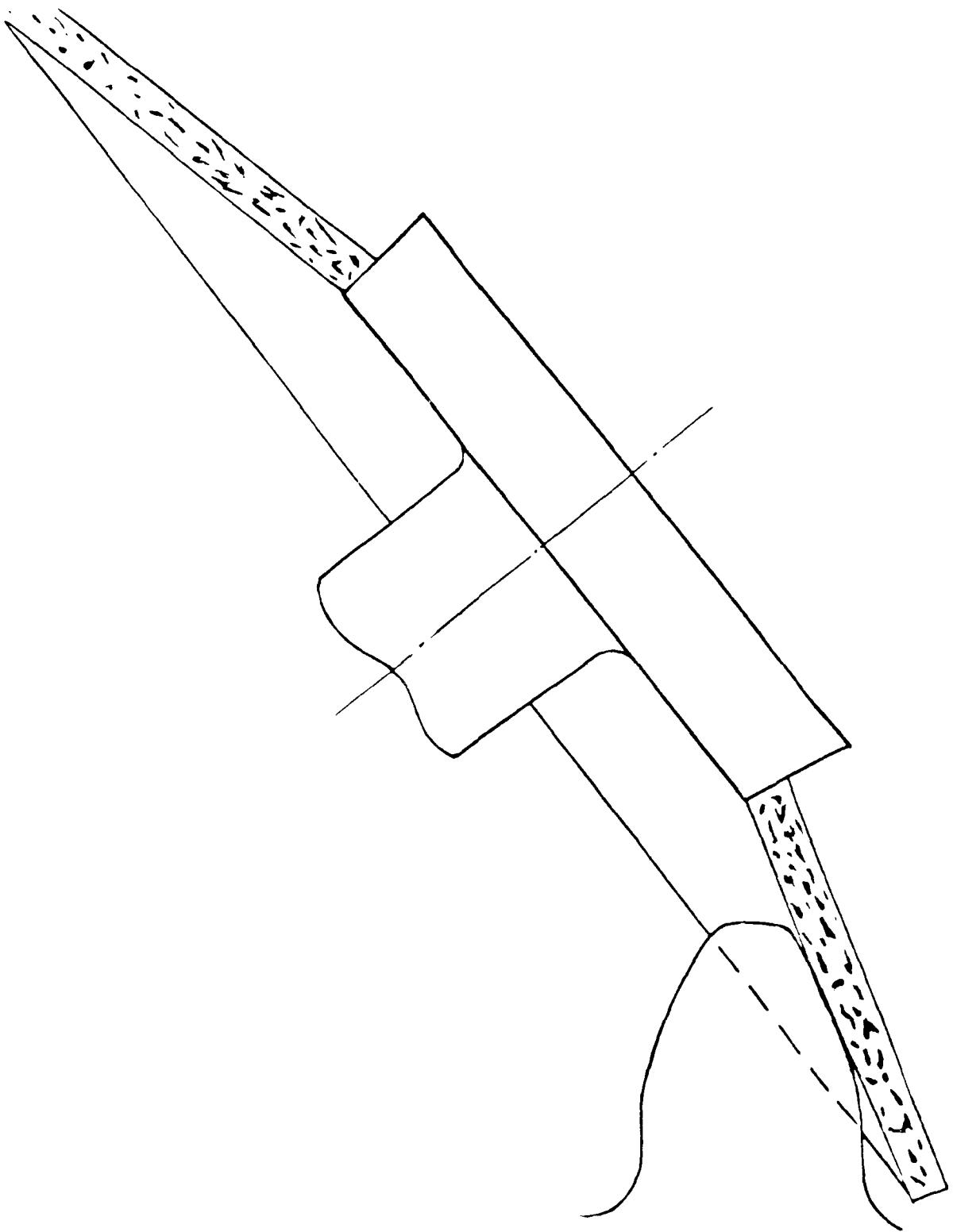


FIG. 1.4

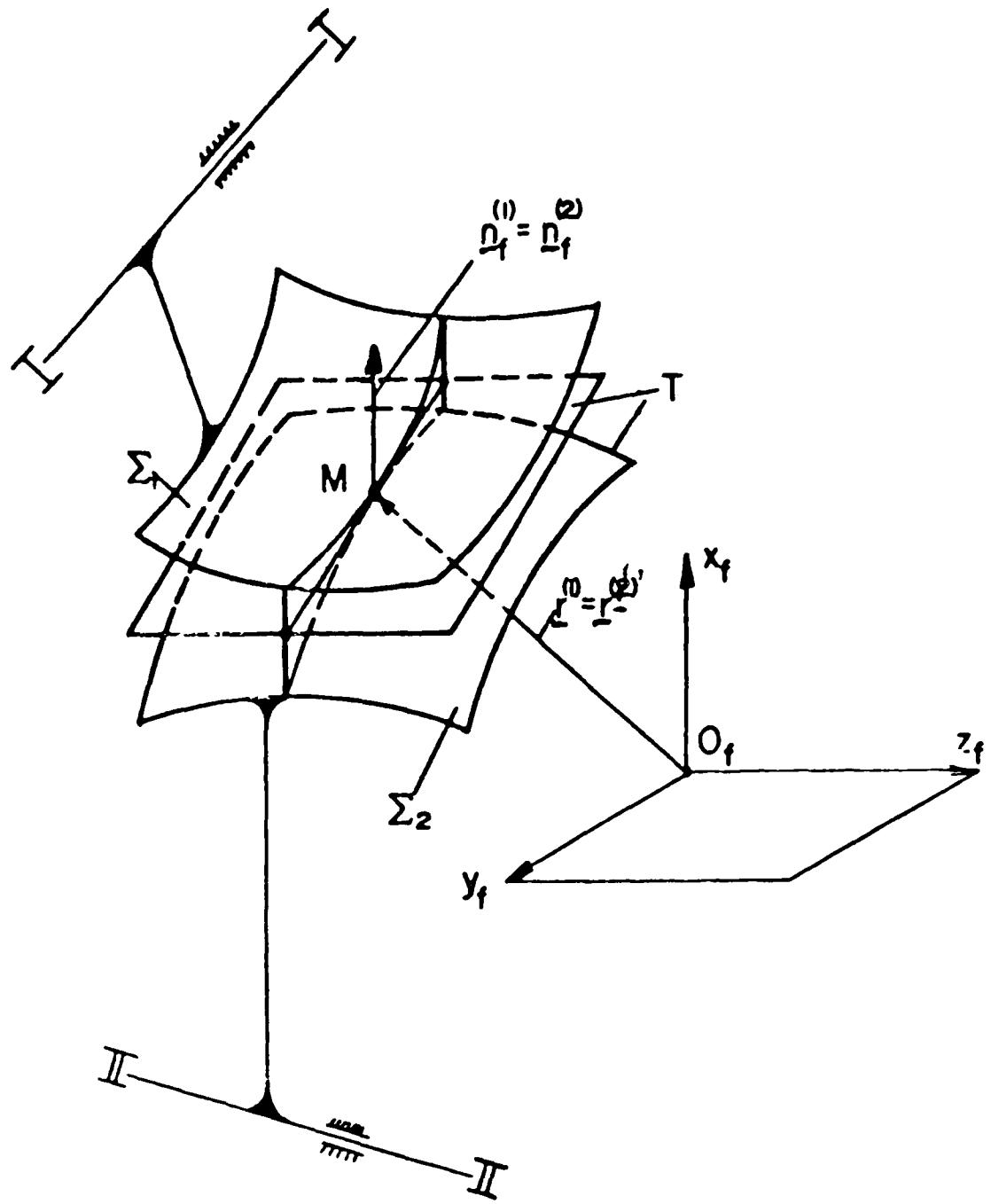


FIG. 1.5

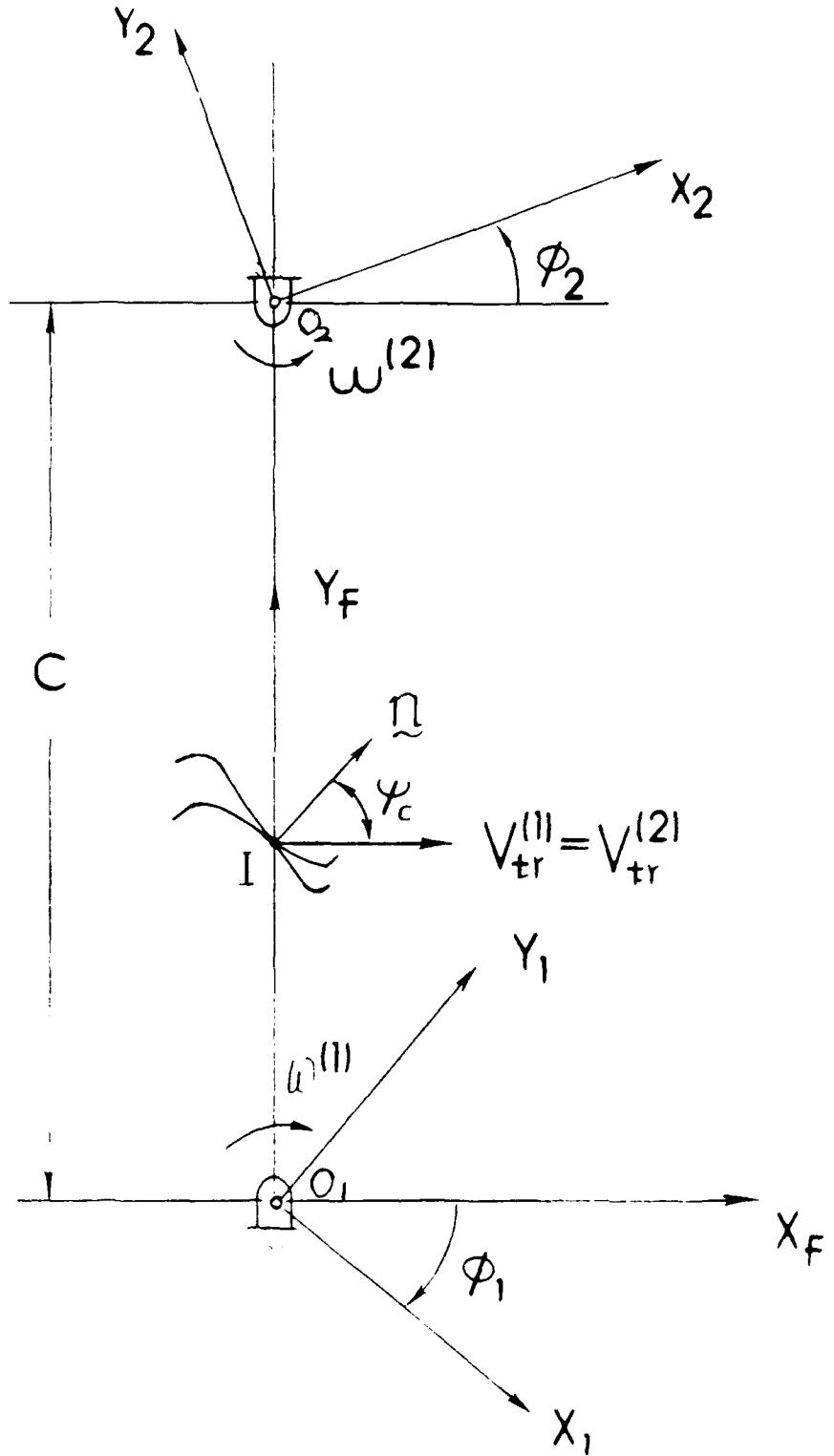


FIG. 2.2.1

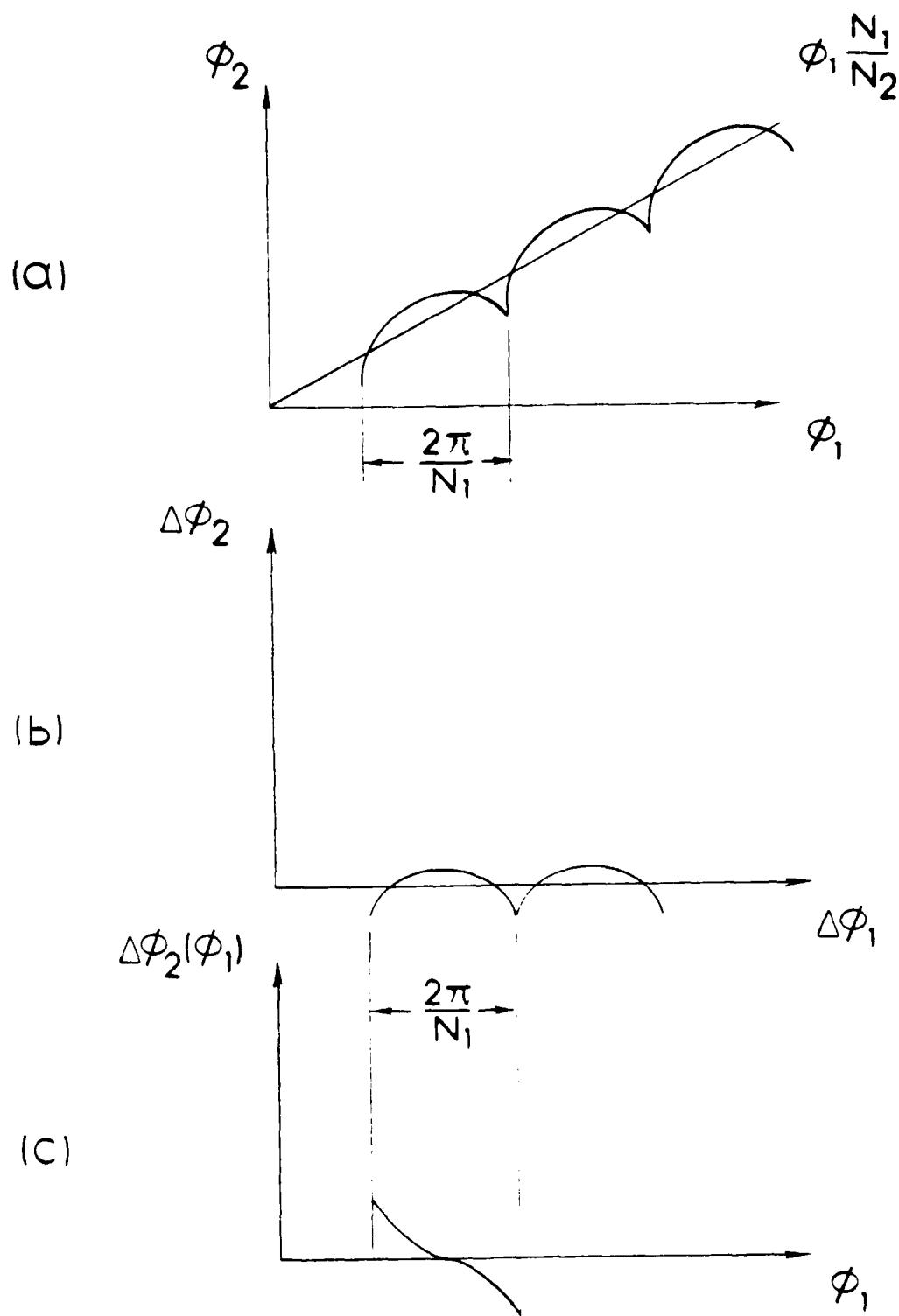


FIG.2.2.2

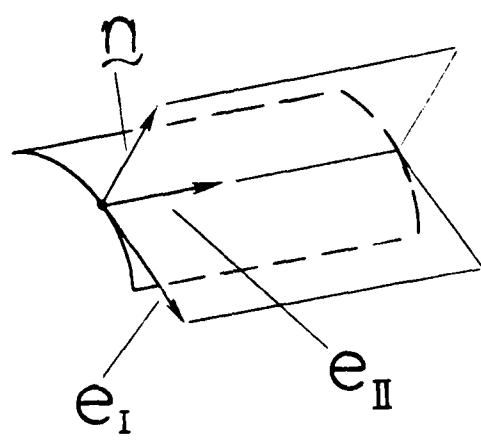


FIG. 2.3.1

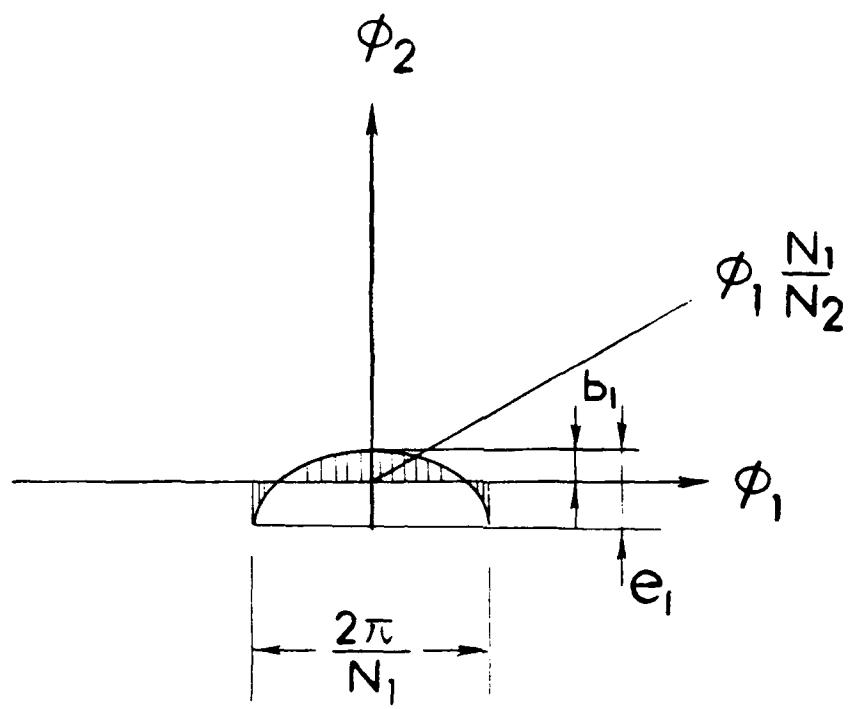


FIG. 3.1.1

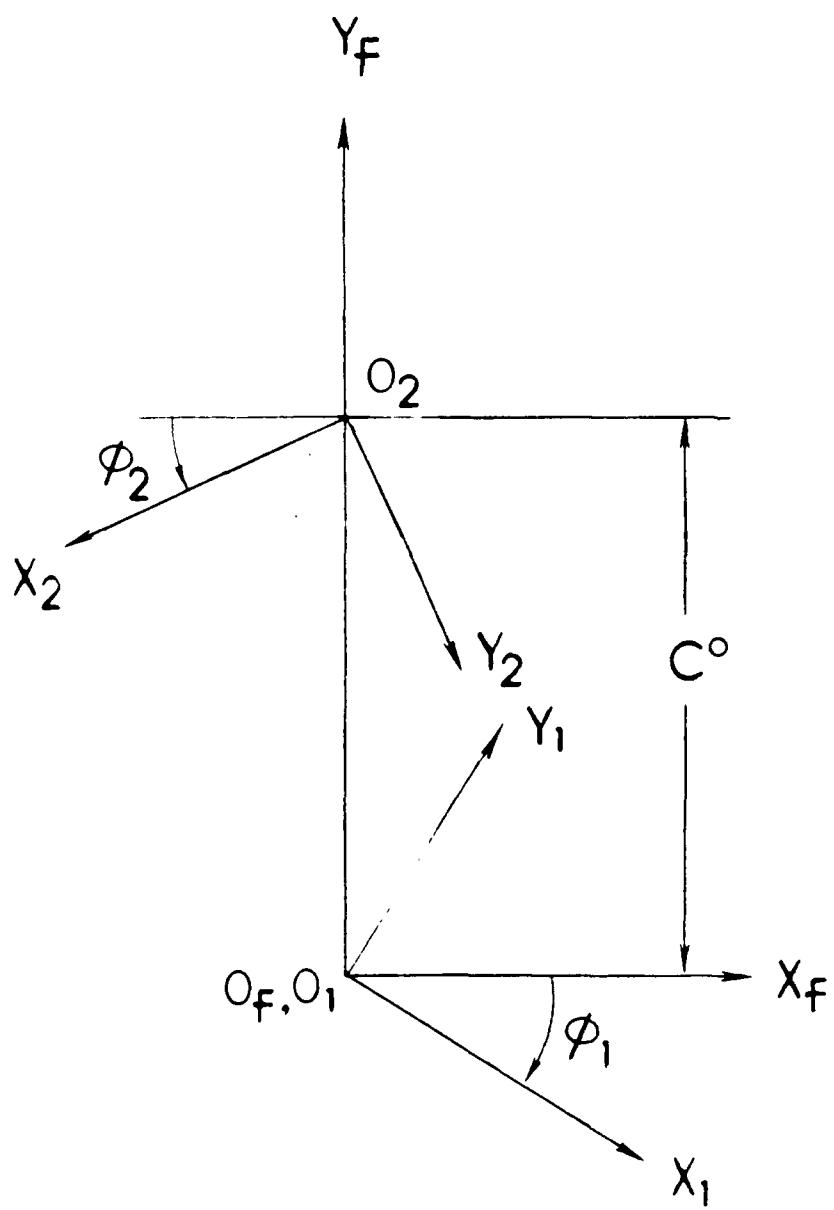


FIG. 3.1.2

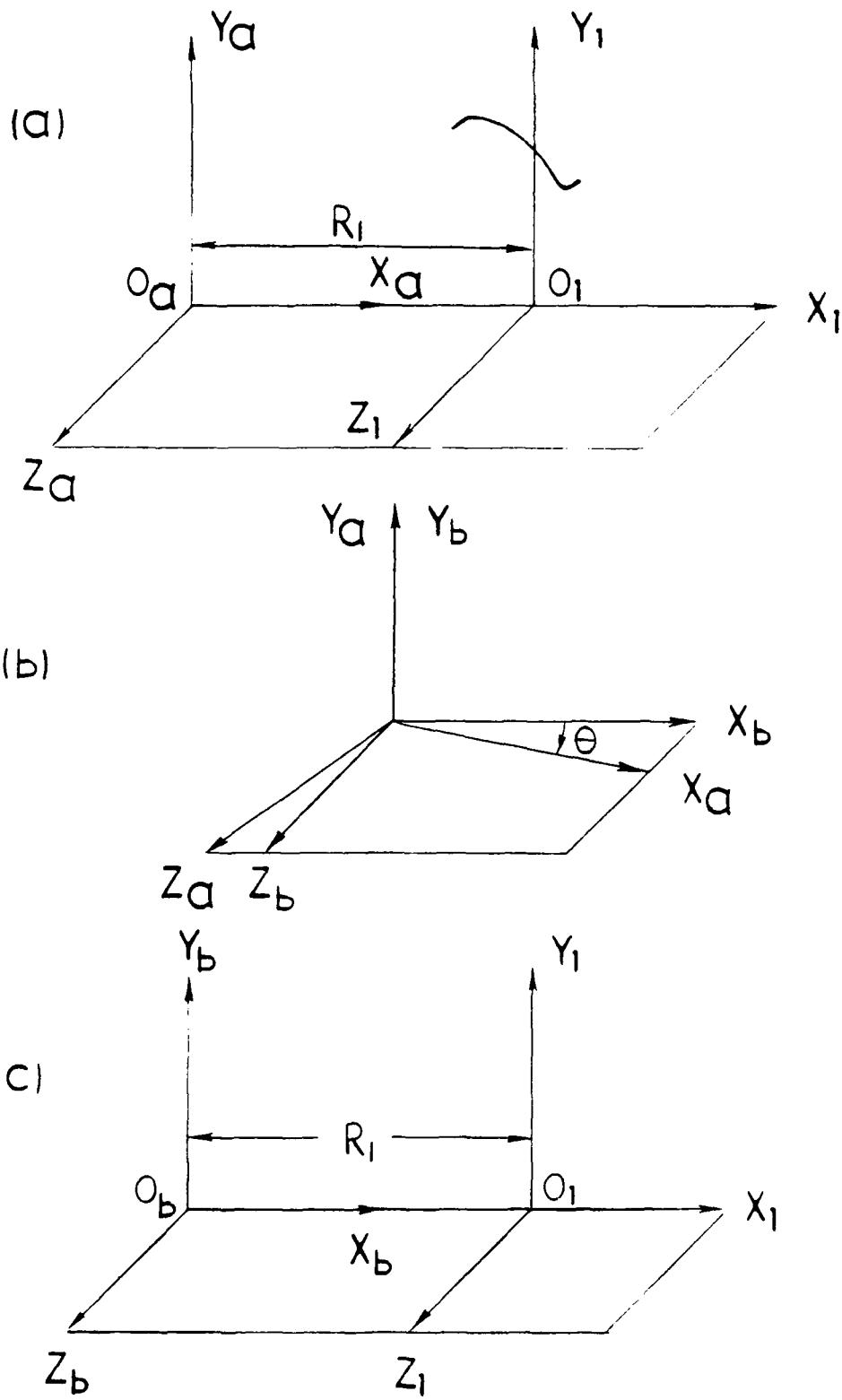


FIG. 3.2.1

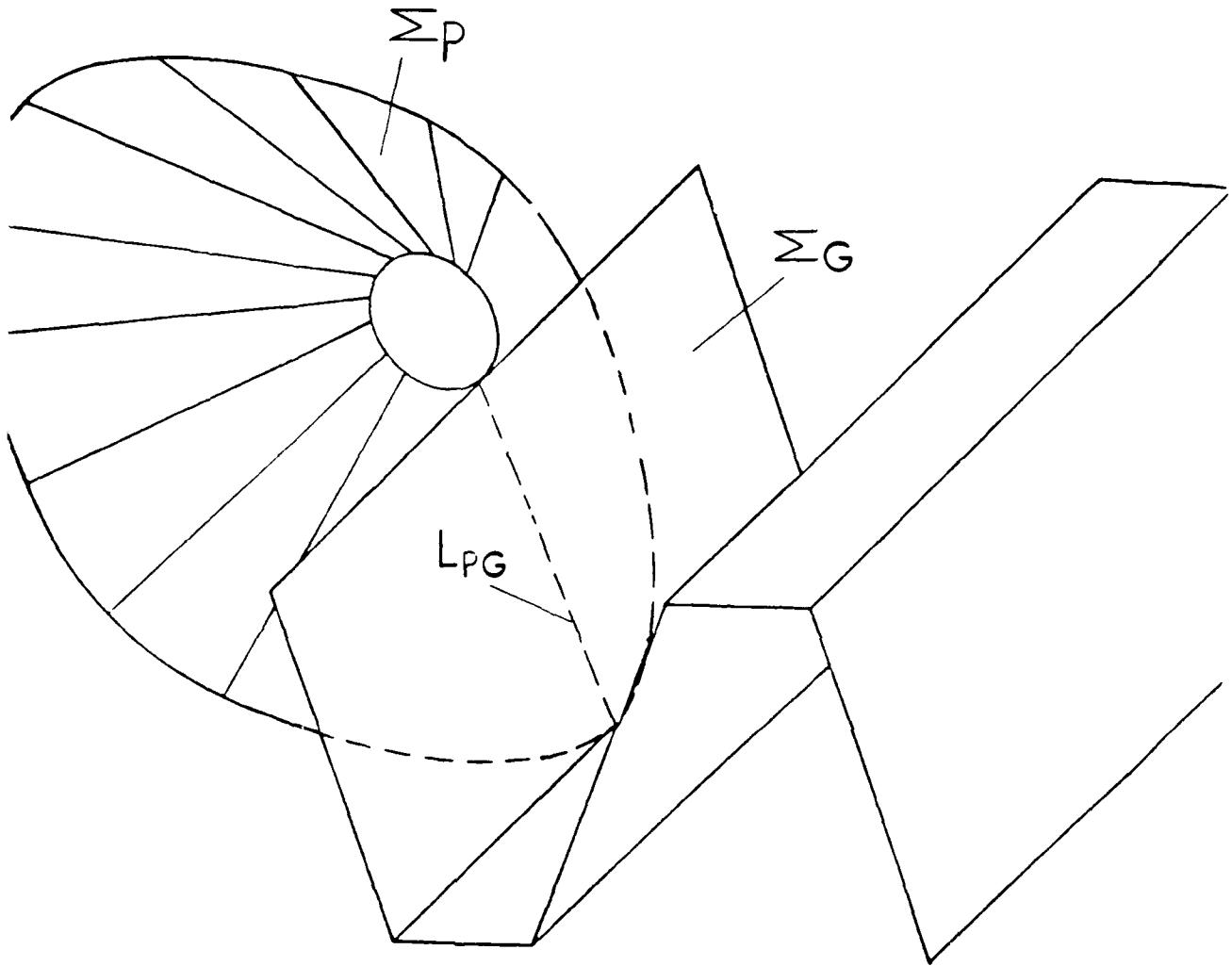


FIG.4.2.O

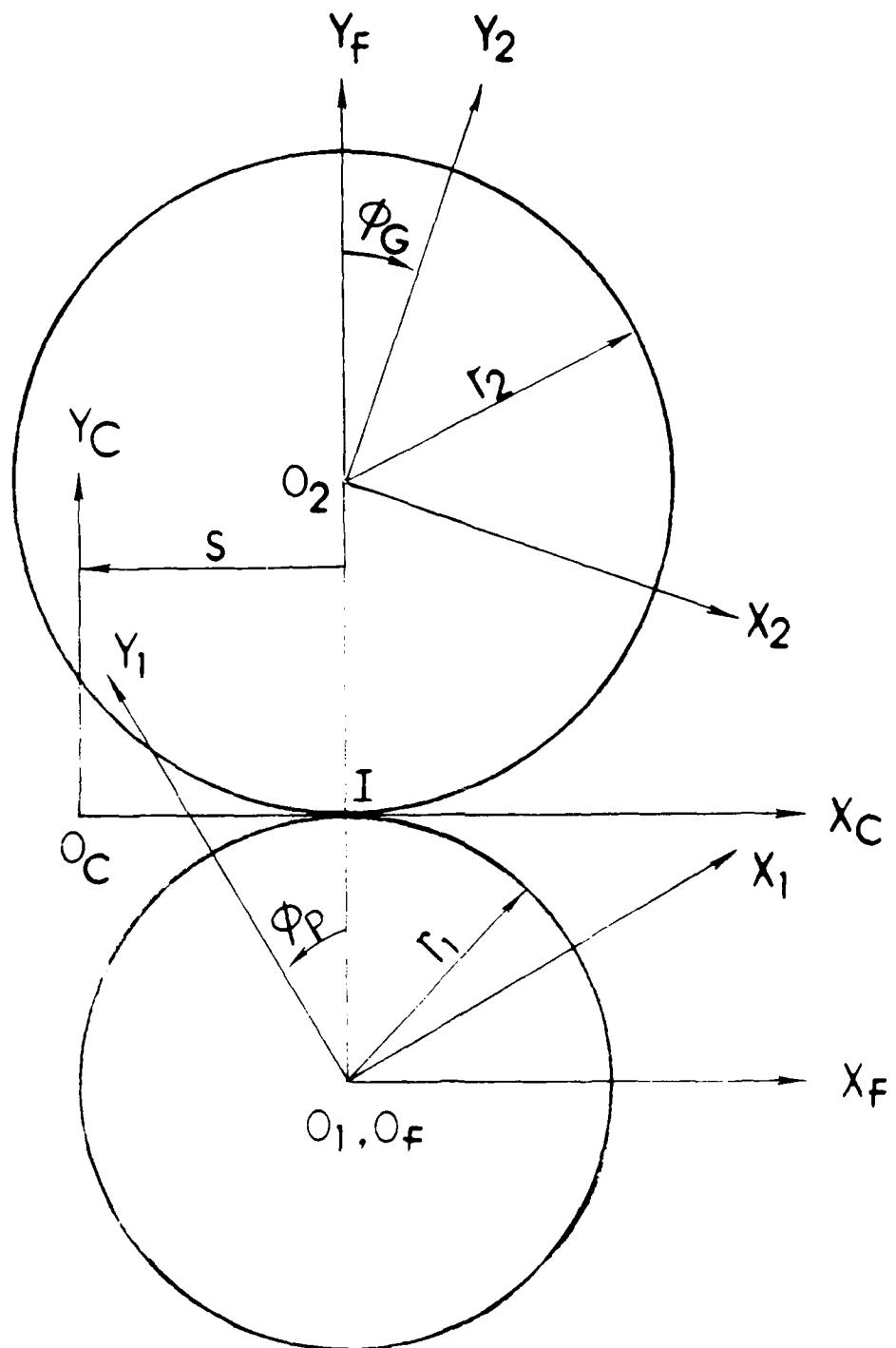


FIG.4.2.1

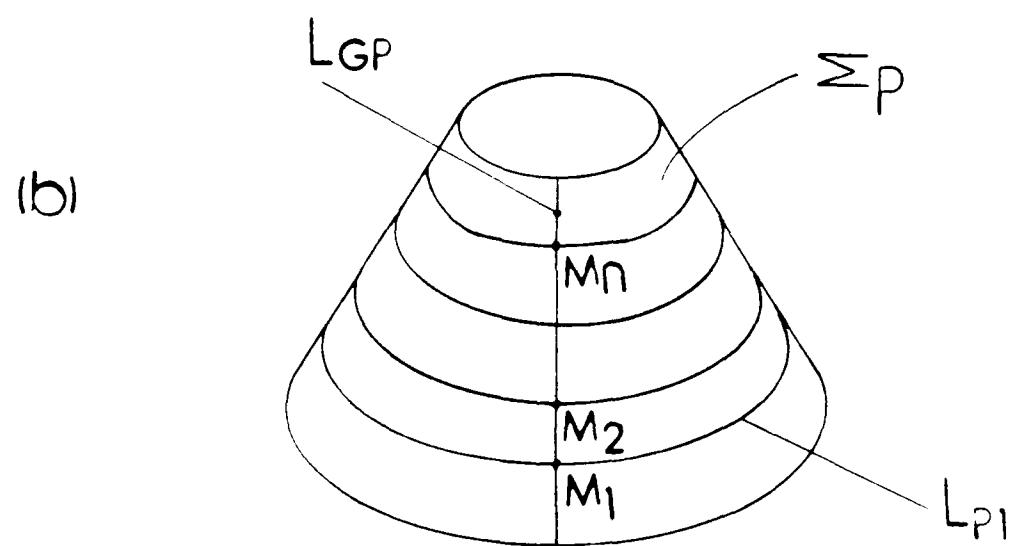
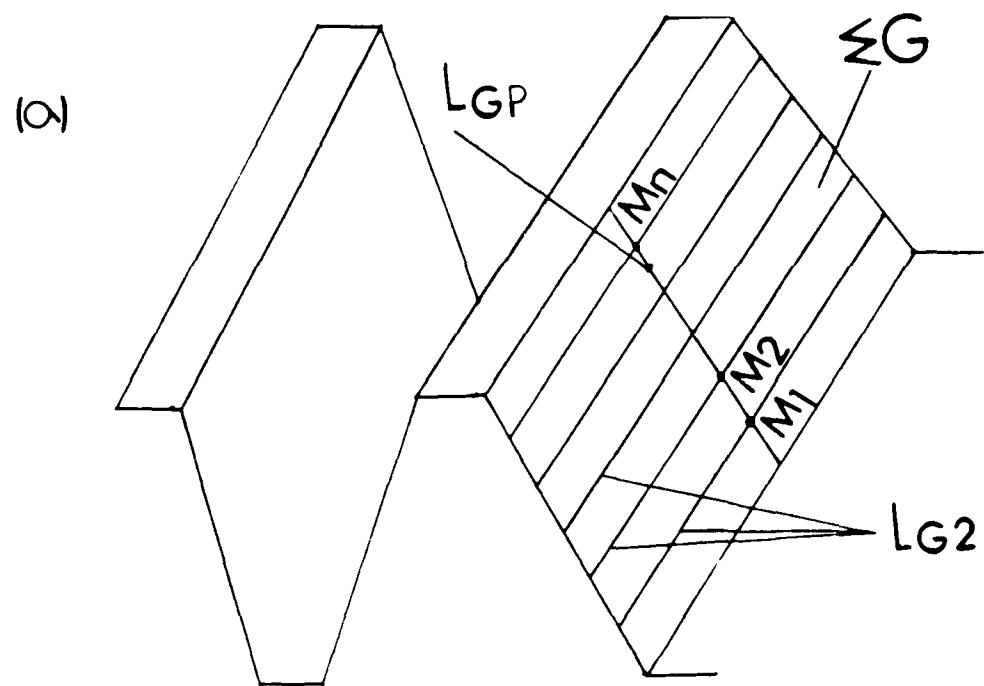


FIG.4.2.2

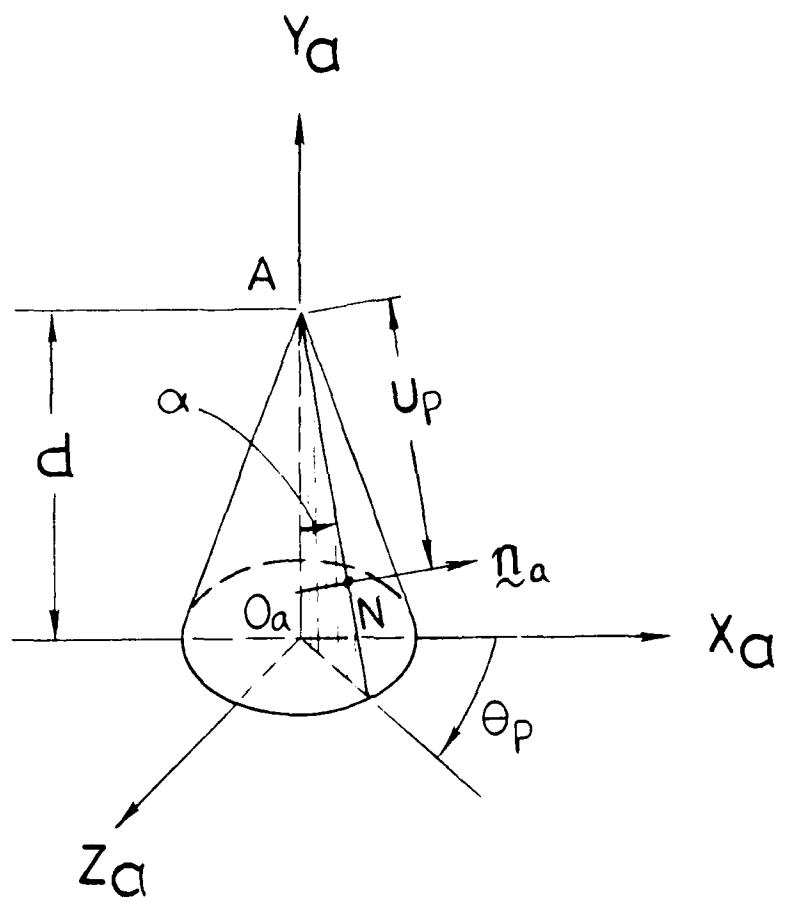


FIG. 4.3.1

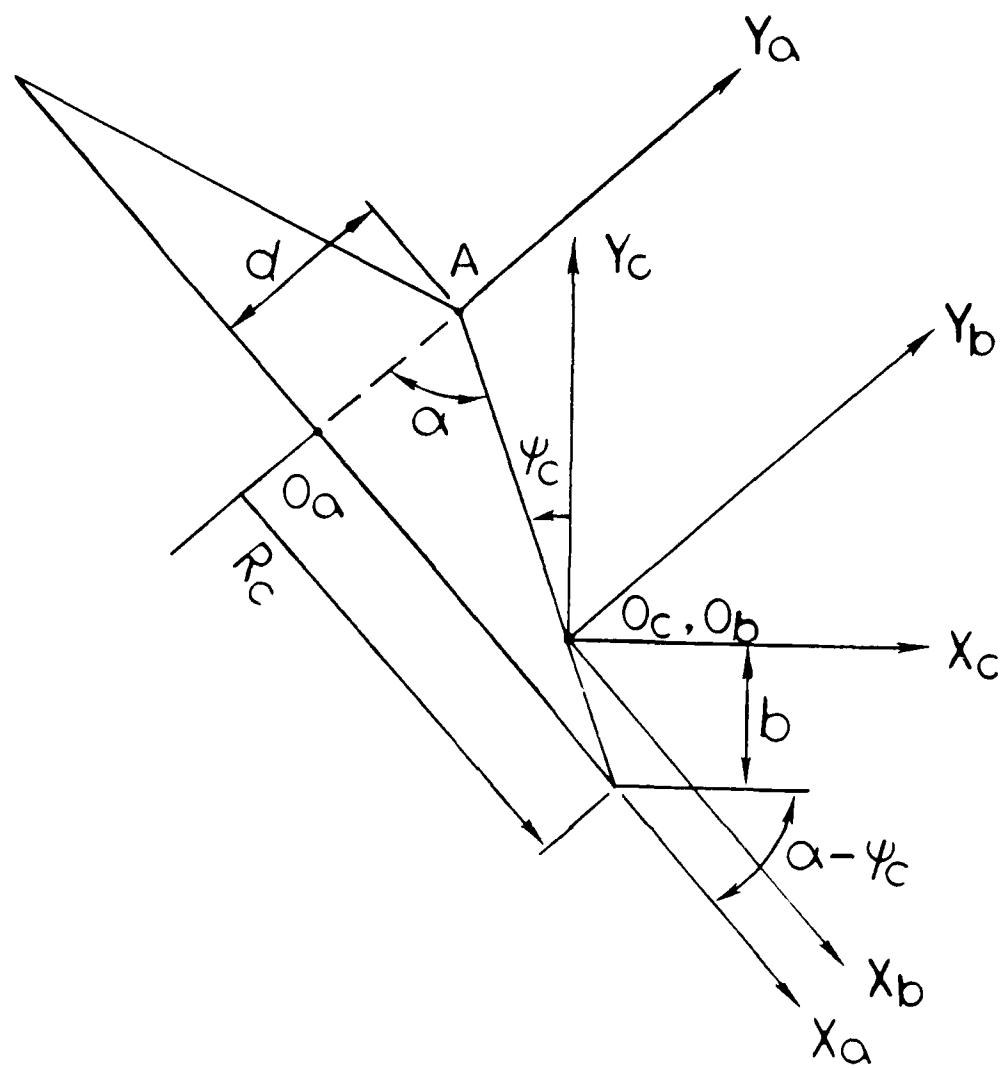


FIG. 4.3.2

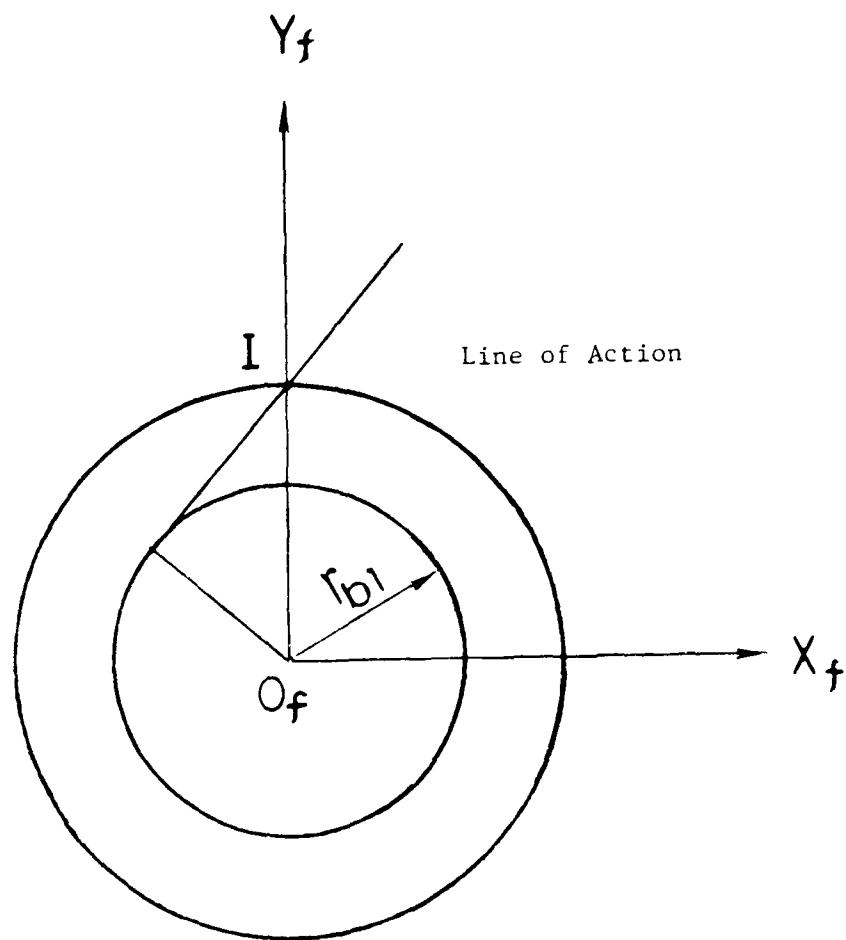


FIG. 4.4.1

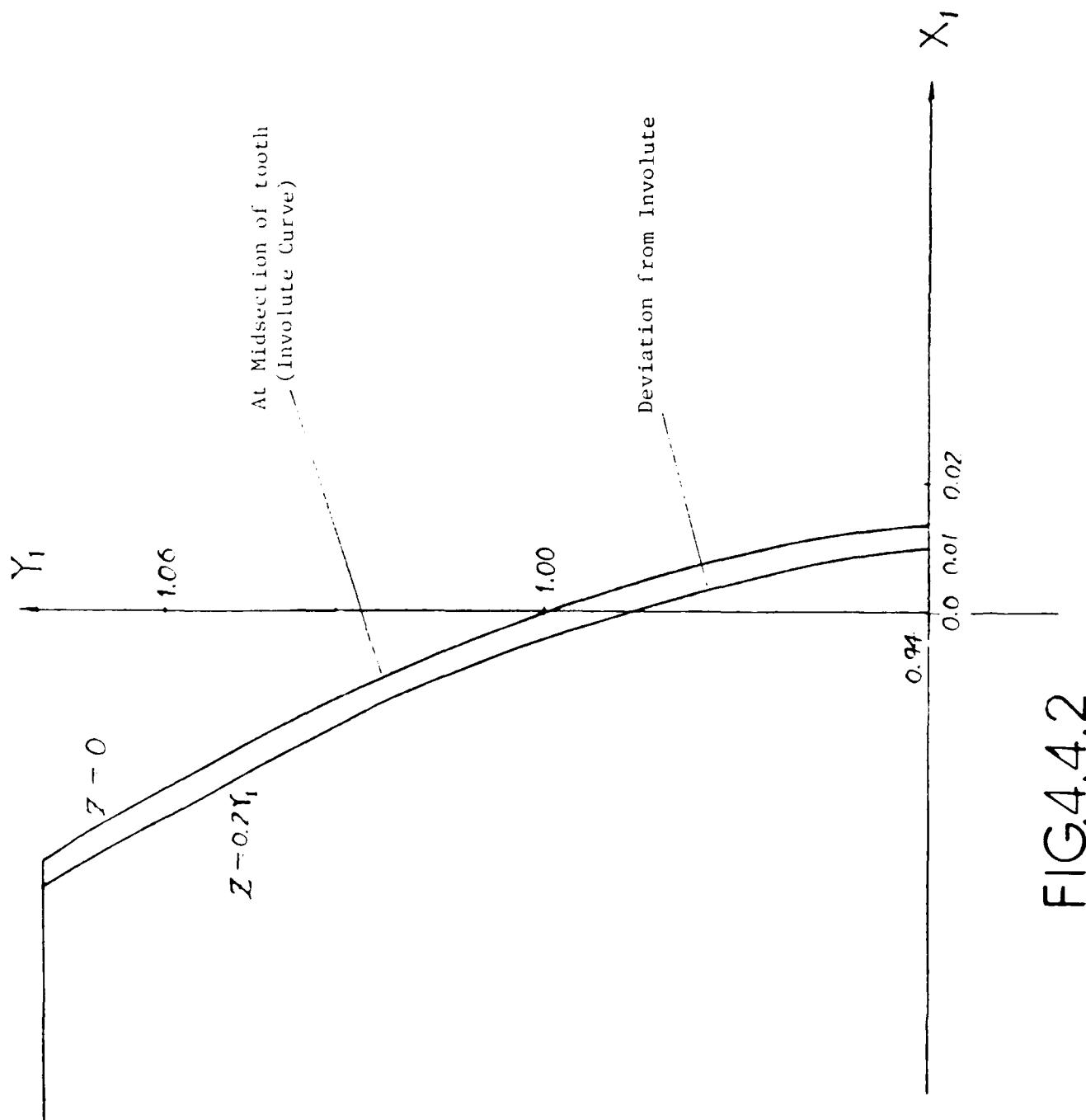


FIG.4.4.2

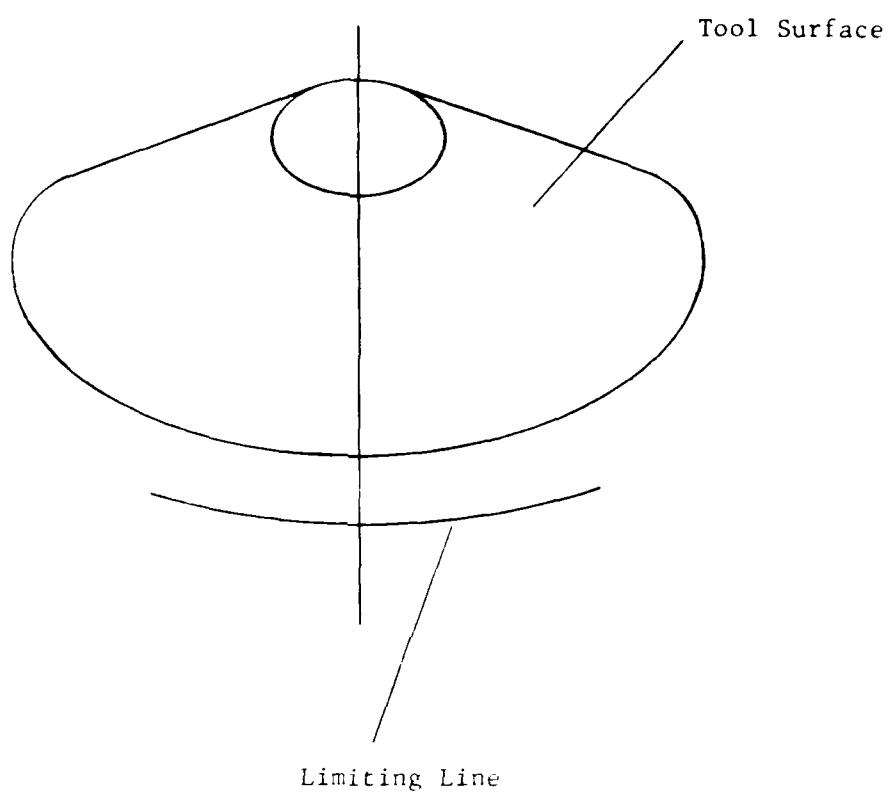


FIG. 4.5.1

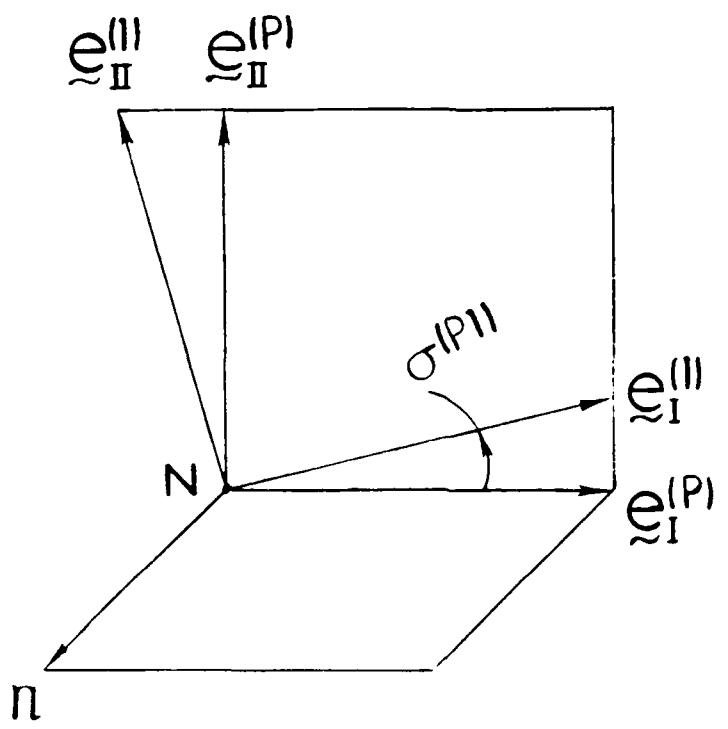


FIG.4.6.1

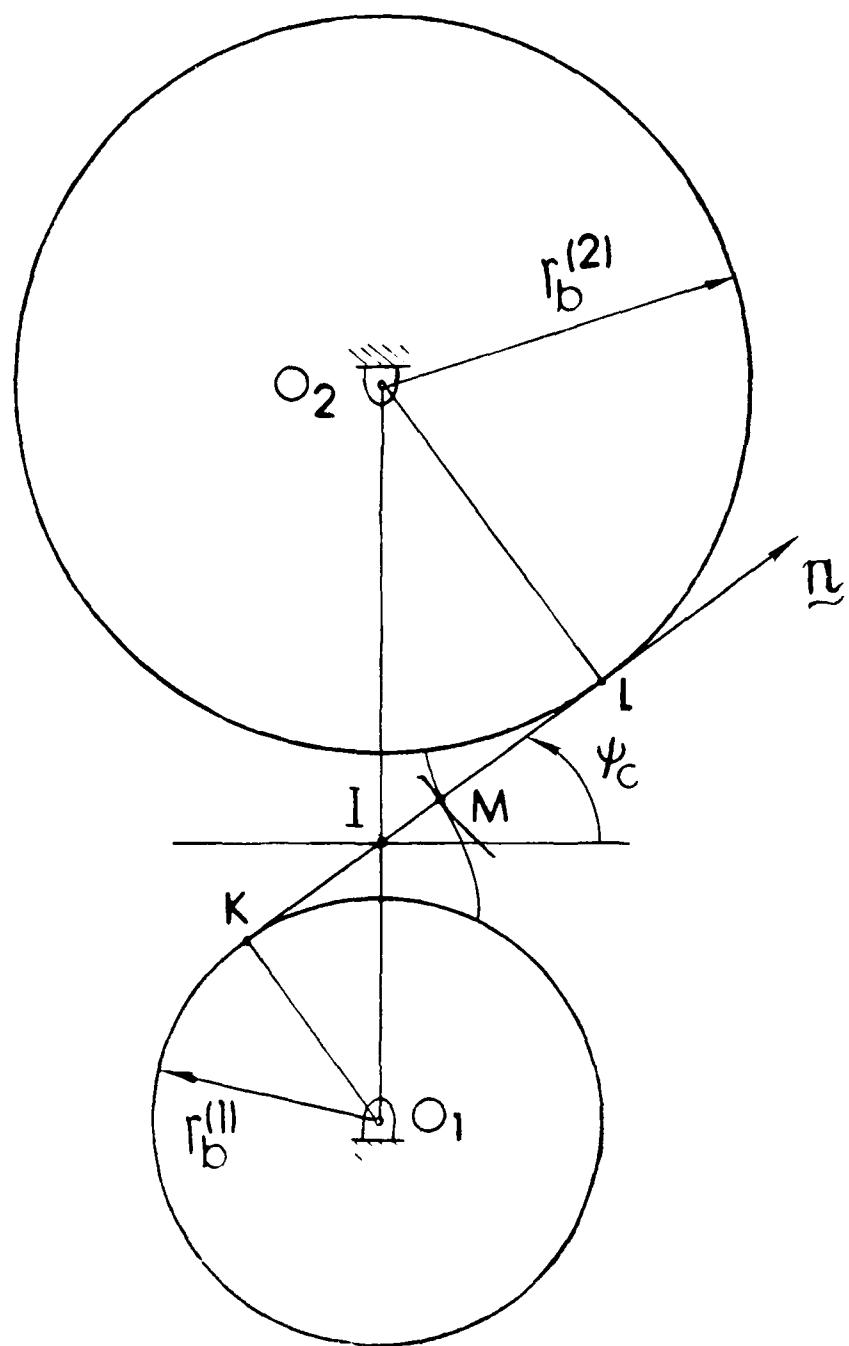


FIG.4.6.2

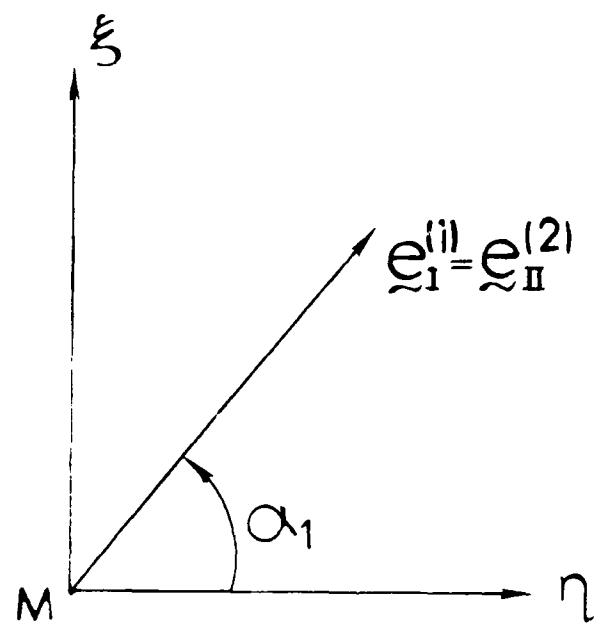


FIG. 4.7.1

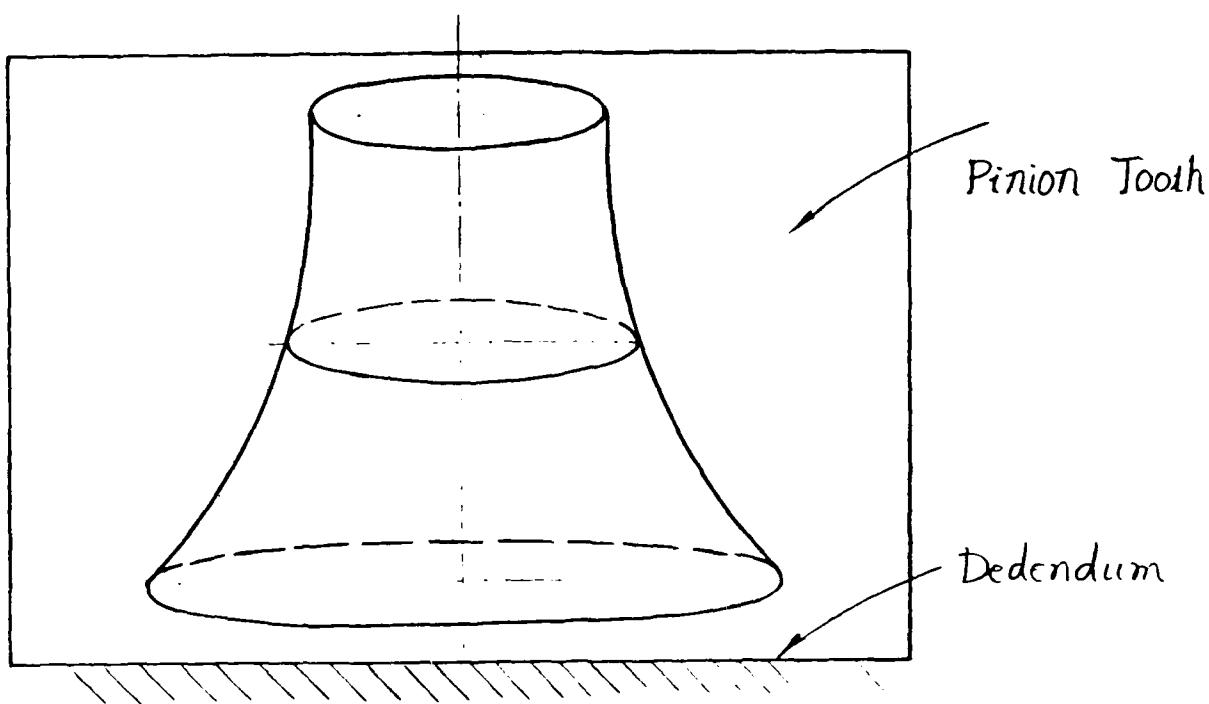


FIG.4.7.2

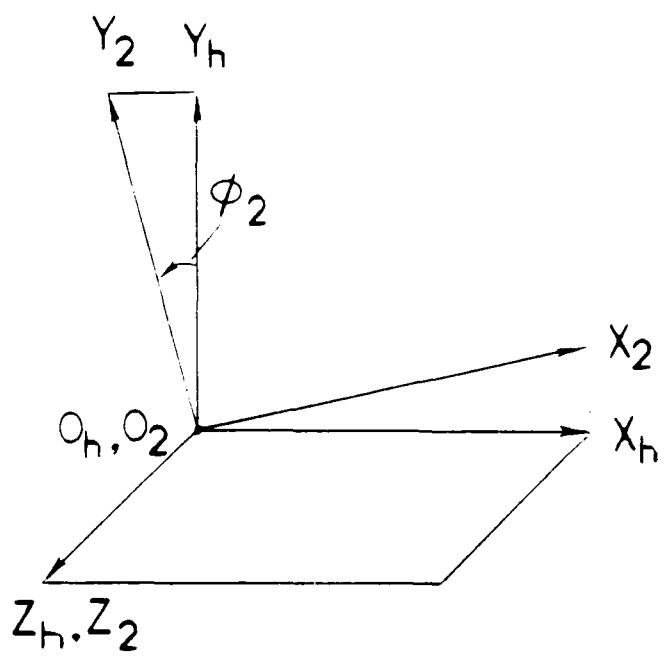
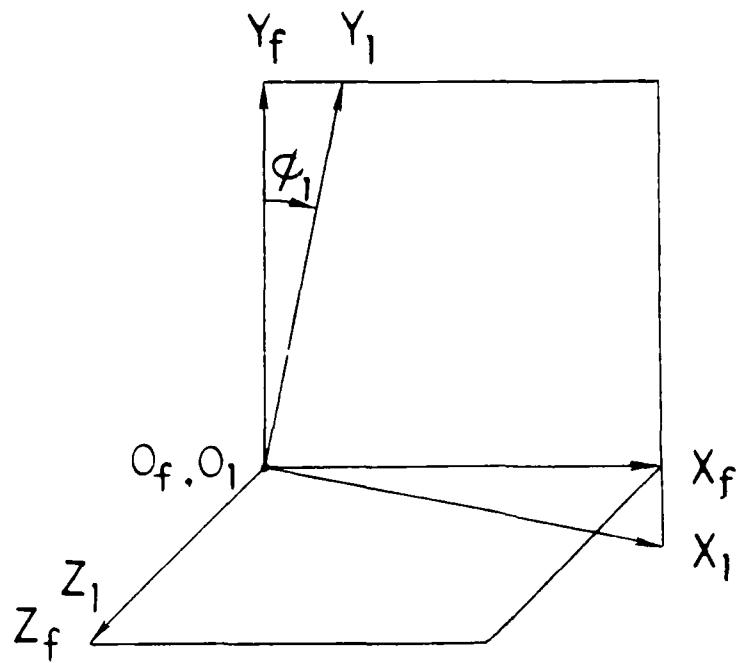


FIG.4.8.1

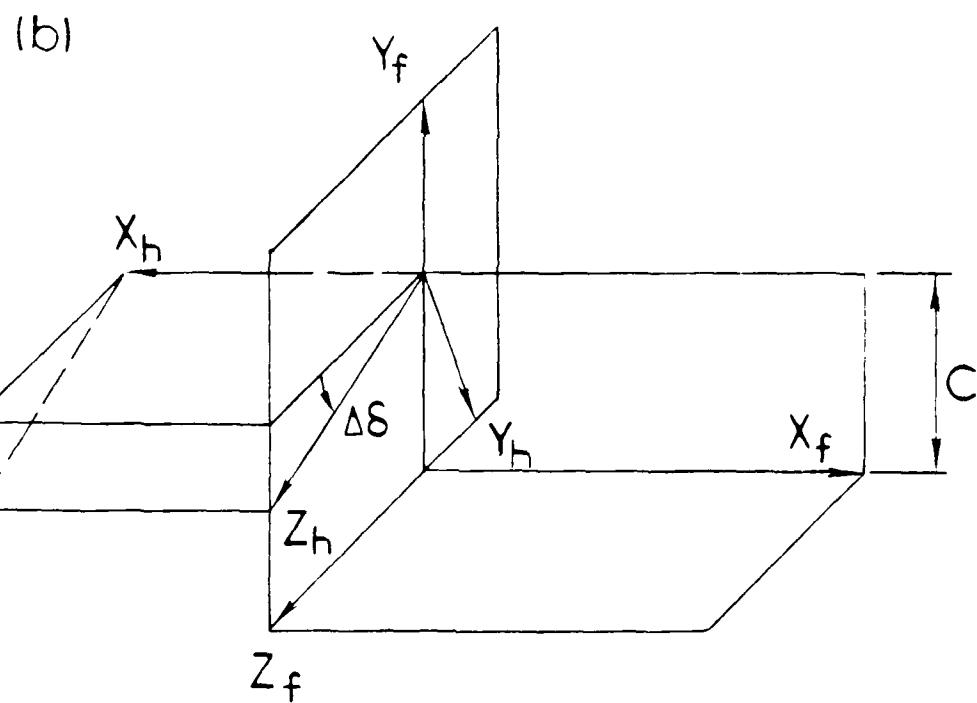
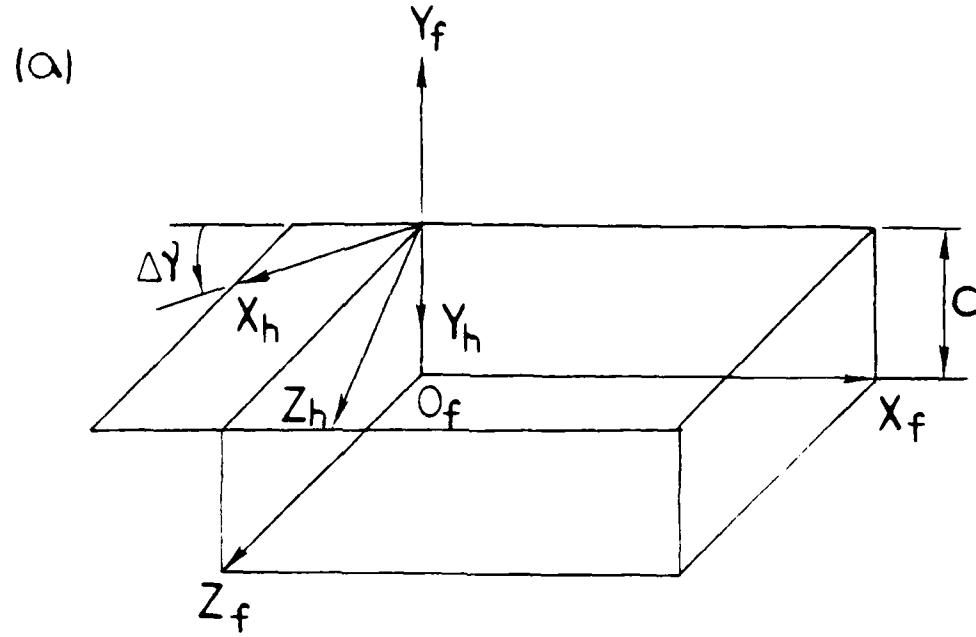


FIG.4.8.2

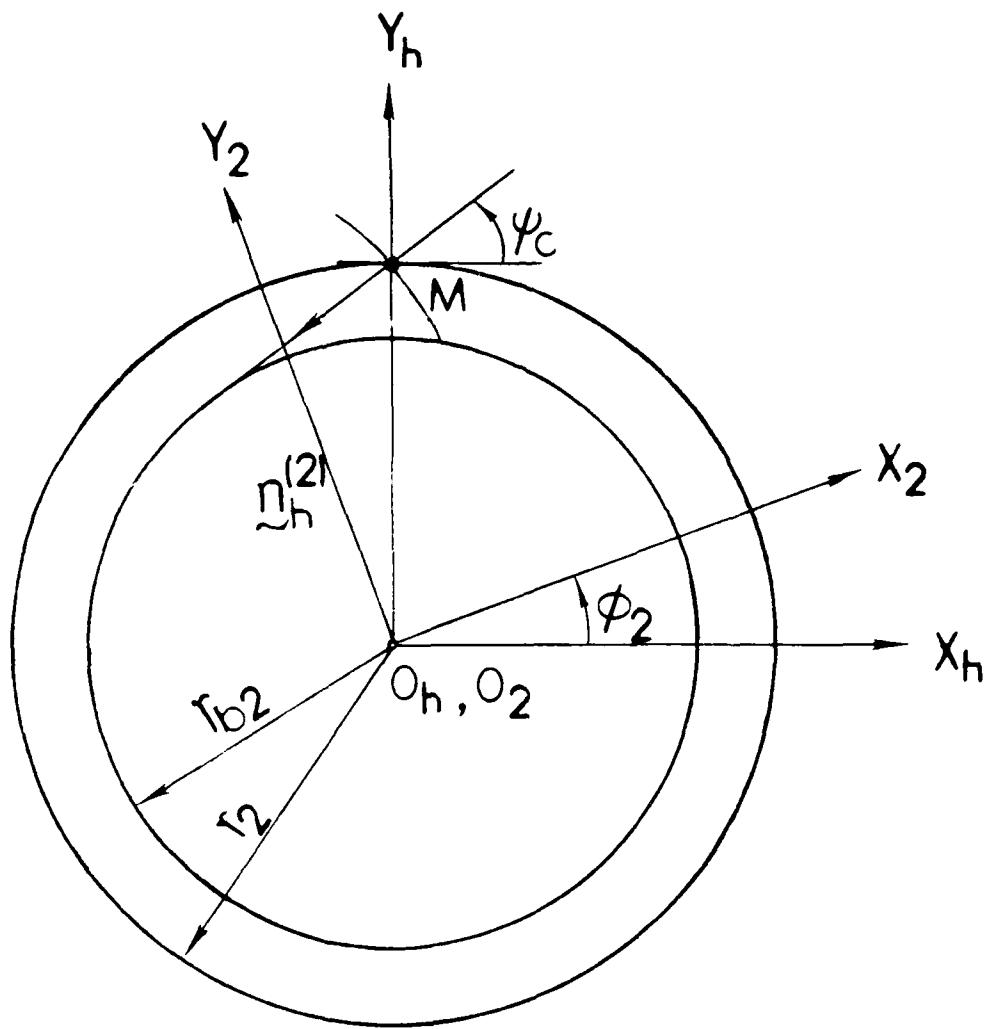


FIG. 4.8.3

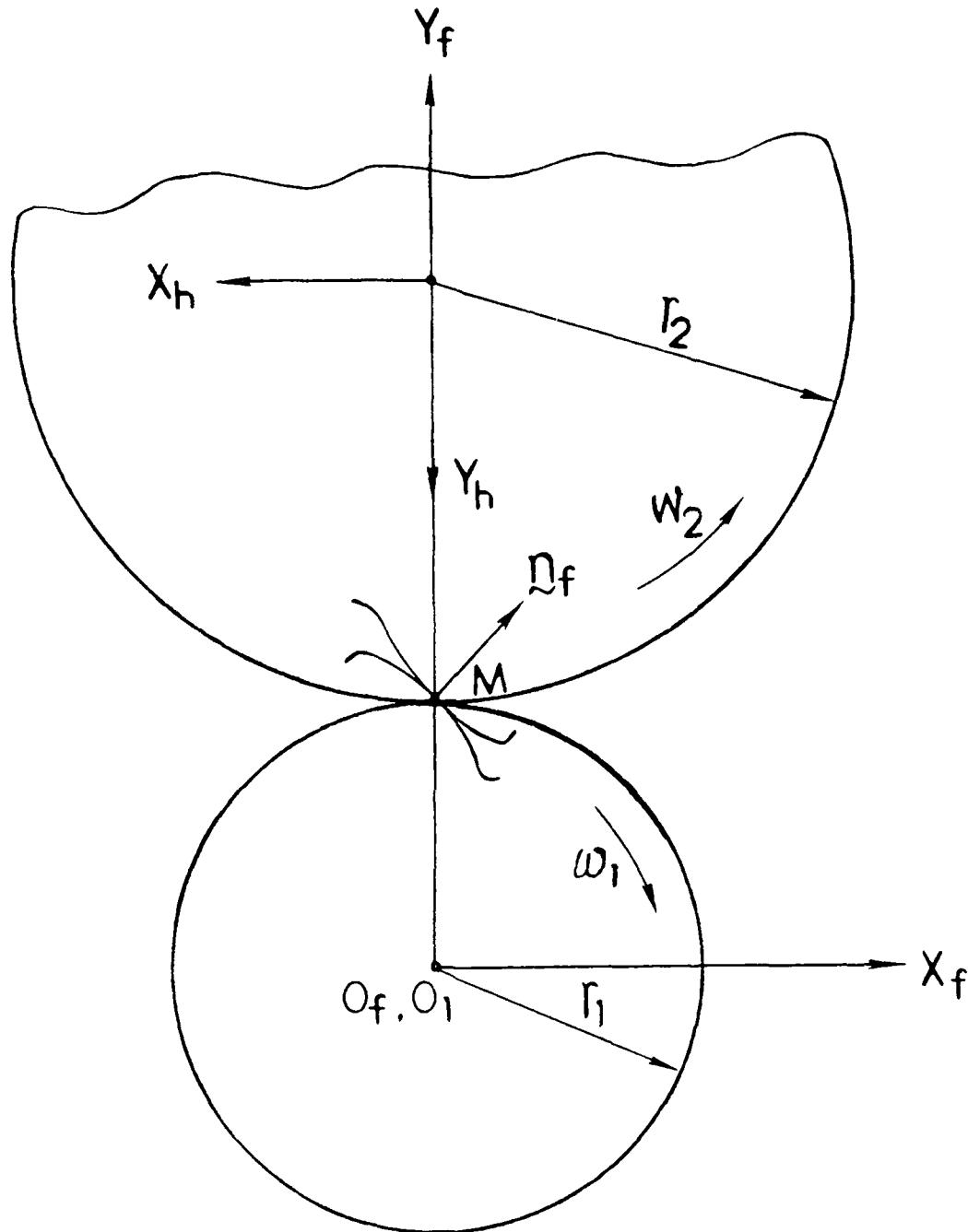


FIG. 4.8.4

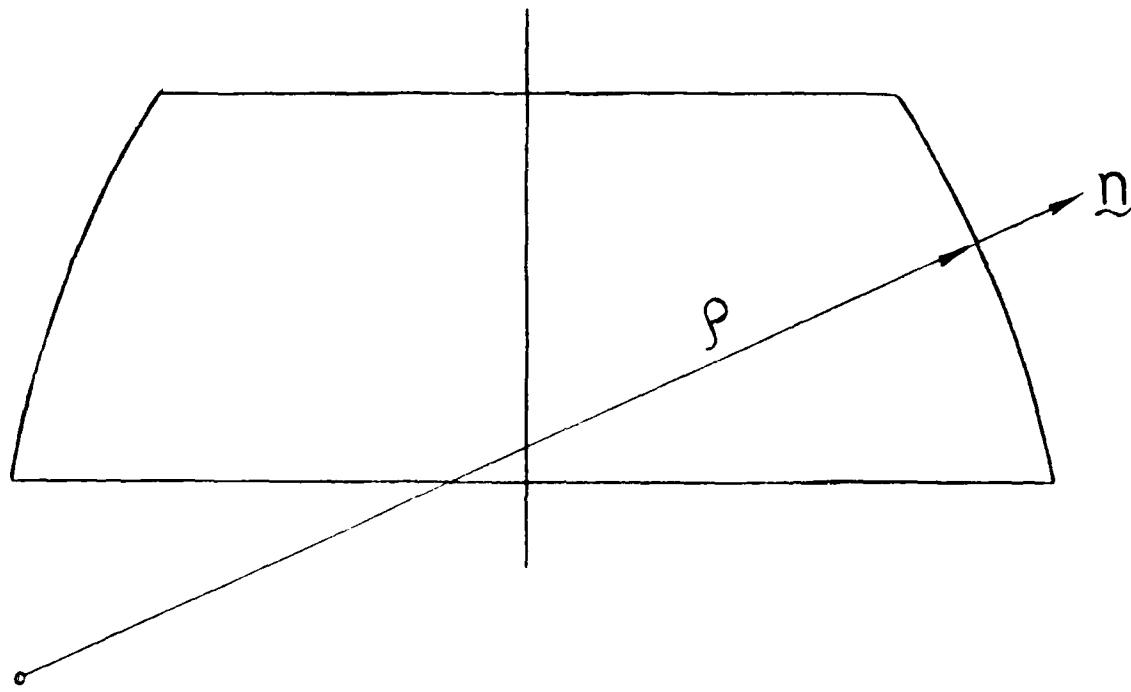


FIG.4.9.1

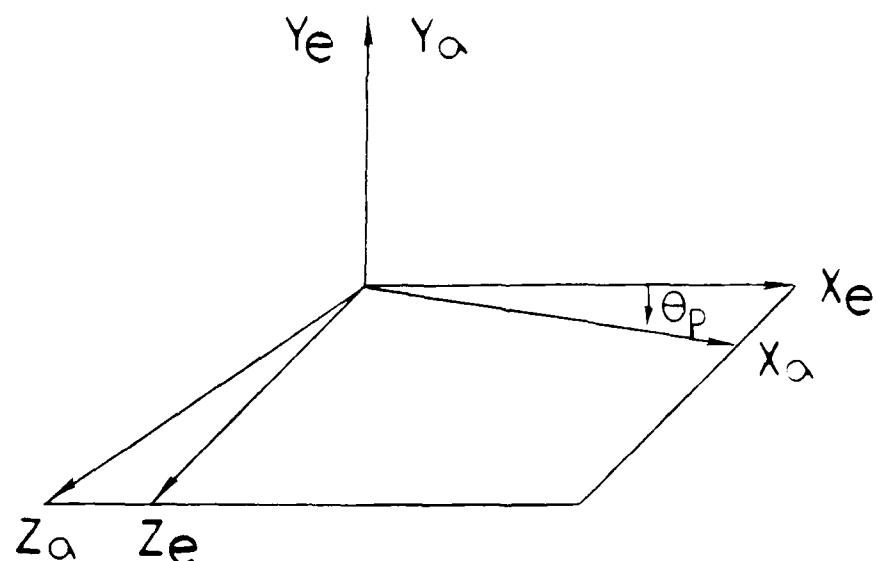
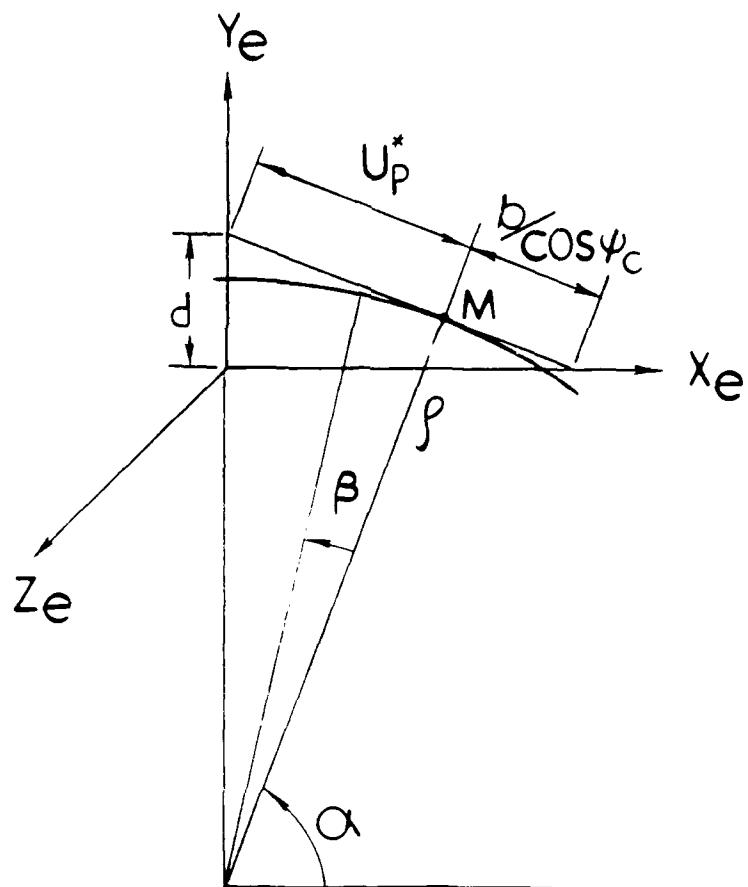


FIG.4.9.2

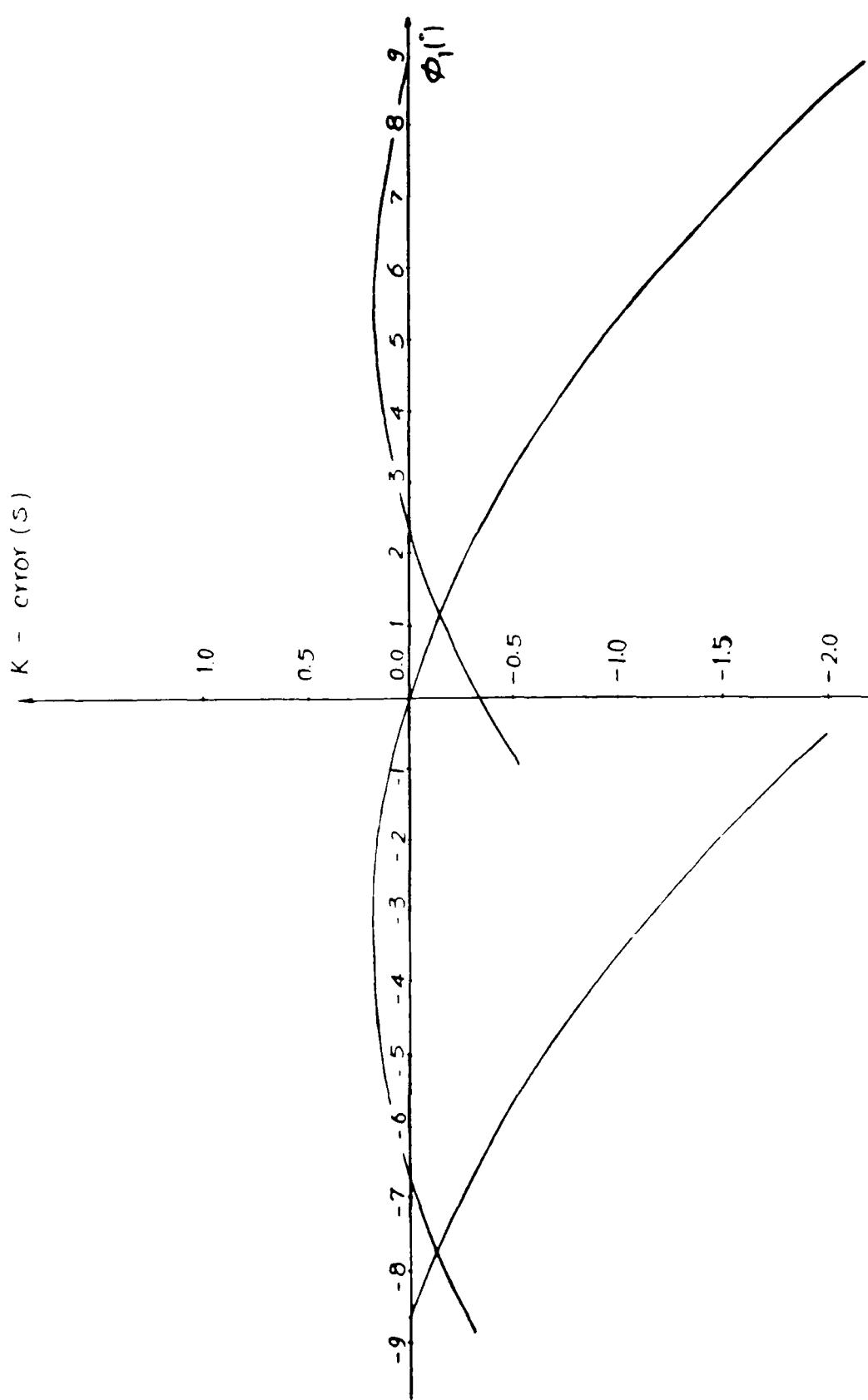


FIG. 4.9.3



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## Report Documentation Page

1. Report No. NASA CR-4135 AVSCOM-TR-88-C-002	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle  Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program		5. Report Date April 1988	
		6. Performing Organization Code	
7. Author(s)  Faydor L. Litvin and Jiao Zhang		8. Performing Organization Report No. None (E-3940)	
		10. Work Unit No. 1L162209AH76 505-63-51	
9. Performing Organization Name and Address  University of Illinois at Chicago Circle Department of Mechanical Engineering Chicago, Illinois 60680		11. Contract or Grant No. NAG3-655	
12. Sponsoring Agency Name and Address  Propulsion Directorate U.S. Army Aviation Research and Technology Activity—AVSCOM Cleveland, Ohio 44135-3191 and NASA Lewis Research Center Cleveland, Ohio 44135-3191		13. Type of Report and Period Covered Contractor Report Final	
15. Supplementary Notes  Project Manager, Robert F. Handschuh, Propulsion Directorate, U.S. Army Aviation Research and Technology Activity—AVSCOM, Lewis Research Center.		14. Sponsoring Agency Code	
16. Abstract  The contents of this report covers: (i) development of optimal geometry for crowned spur gears; (ii) methods for their generation; and (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned spur gears. The developed method for synthesis is used for the determination of the optimal geometry for crowned pinion surface and is directed to reduce the sensitivity of the gears to misalignment, localize the bearing contact, and guarantee the favorable shape and low level of the transmission errors. A new method for the generation of the crowned pinion surface has been proposed. This method is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. The crowned pinion surface can be also generated by a generating plane whose motion is provided by an automatic grinding machine controlled by a computer. The TCA program simulates the meshing and bearing contact of the misaligned gears. The transmission errors are also determined.			
17. Key Words (Suggested by Author(s))  Spur gears Gears Tooth contact analysis Crowned spur gears		18. Distribution Statement  Unclassified—Unlimited Subject Category 37	
19. Security Classif. (of this report)  Unclassified	20. Security Classif. (of this page)  Unclassified	21. No of pages 134	22. Price* A07

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